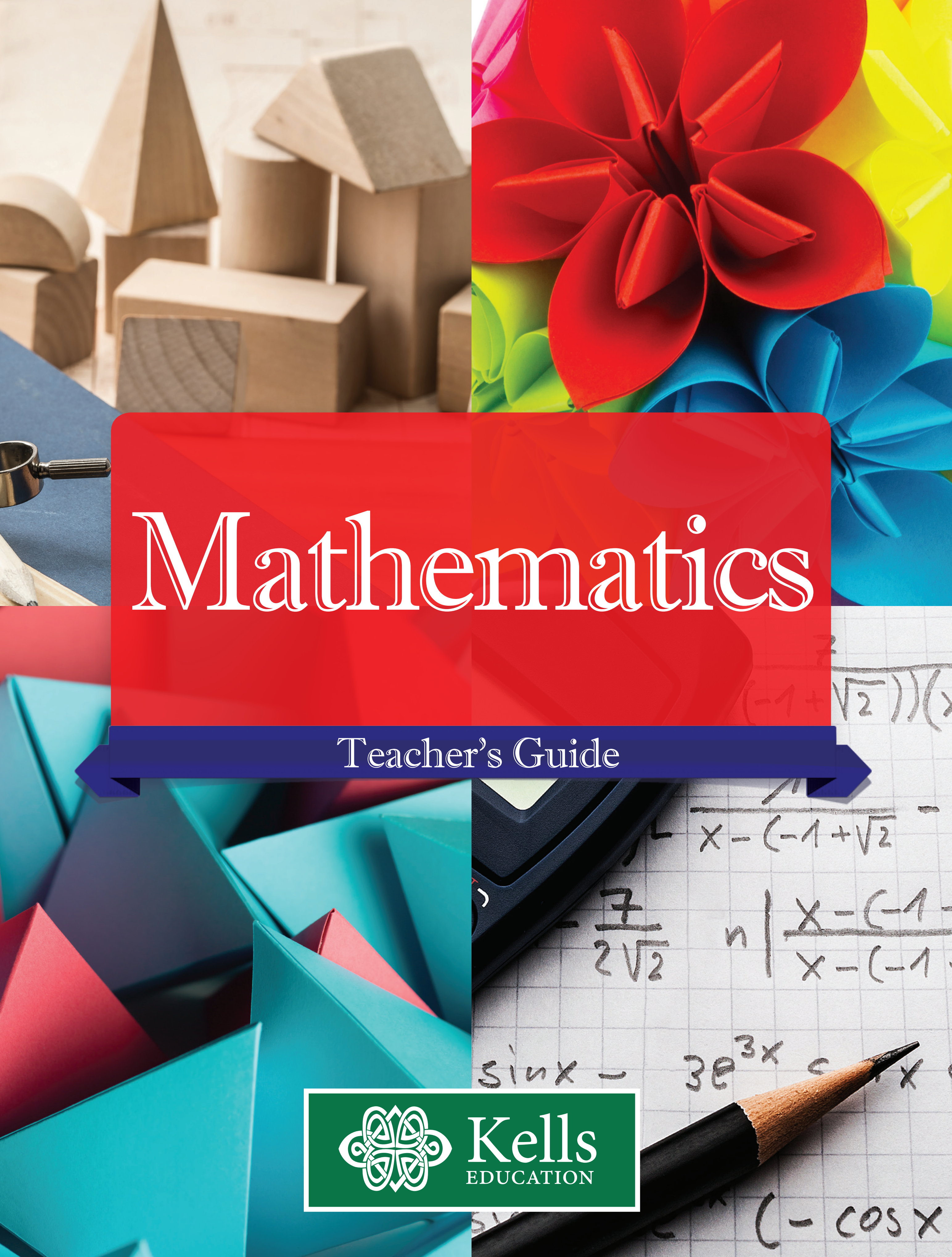

Mathematics

Teacher's Guide

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Mathematics Teacher's Guide

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Printed in Mexico

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To the Teacher

Dear Teacher

Teaching mathematics has always been challenging, but nowadays, with the growing technology invasion, it is even more difficult because students are used to getting immediate results without analysis or validity checking.

This book has the objective of accompanying your teaching through activities that promote mathematical skills, which are paramount to succeed in the international mathematics assessments: reading, verbal-linguistic, mathematizing, reasoning and strategic skills.

There are recommendations of the best teaching practices in mathematics as well as the description of the skills that students will be using during specific sessions so as to help you develop argumentative didactic planners.

We are certain that through guidance and leadership students will learn and enjoy the activities we offer in this book. And you too will enjoy facilitating mathematical content.

The authors

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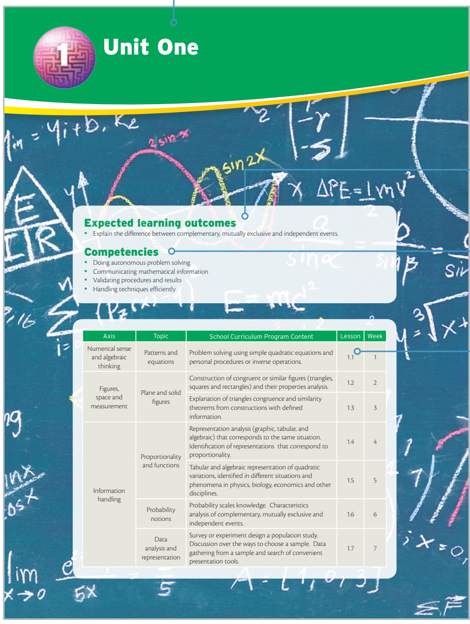
Table of Contents

How to use this book

In order to know how to use this book in detail, please take a look at the sections which conform the text:

General structure



Unit One

Expected learning outcomes

- Explain the difference between complementary, mutually exclusive and independent events.

Competencies

- Doing autonomous problem solving
- Communicating mathematical information
- Validating procedures and results
- Handling techniques efficiently

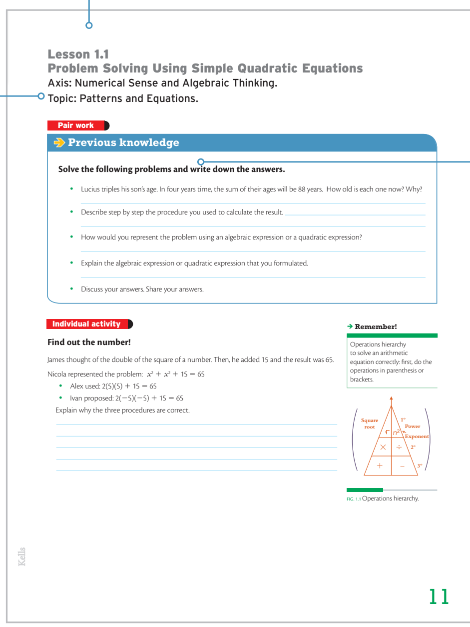
Axis	Topic	School Curriculum Program Content	Lesson	Week
Numerical sense and algebraic thinking	Patterns and equations	Problem solving using simple quadratic equations and personal procedures or inverse operations.	1.1	1
Figures, space and measurement	Plane and solid figures	Construction of congruent or similar figures (triangles, squares and rectangles) and their properties analysis.	1.2	2
		Explanation of triangles congruence and similarity theorems from constructions with defined information.	1.3	3
Information handling	Proportionality and functions	Representation analysis (graphic, tabular, and algebraic) that corresponds to the same situation. Identification of representations that correspond to proportionality.	1.4	4
		Tabular and algebraic representation of quadratic variations, identified in different situations and phenomena in physics, biology, economics and other disciplines.	1.5	5
	Probability notions	Probability scales knowledge. Characteristics analysis of complementary, mutually exclusive and independent events.	1.6	6
	Data analysis and representation	Survey or experiment design a population study. Discussion over the ways to choose a sample. Data gathering from a sample and search of convenient presentation tools.	1.7	7

Unit number: Each unit is divided in lessons.

Expected learning outcomes: What you will be able to do after finishing the unit.

Competencies: The abilities that you will improve.

Table of contents: It includes the offlcial school curriculum program, axis and topic as well as suggested course programming.



Lesson 1.1
Problem Solving Using Simple Quadratic Equations
Axis: Numerical Sense and Algebraic Thinking.
Topic: Patterns and Equations.

Pair work

Previous knowledge

Solve the following problems and write down the answers.

- Lucius triples his son's age. In four years time, the sum of their ages will be 88 years. How old is each one now? Why?
- Describe step by step the procedure you used to calculate the result.
- How would you represent the problem using an algebraic expression or a quadratic expression?
- Explain the algebraic expression or quadratic expression that you formulated.
- Discuss your answers. Share your answers.

Individual activity

Find out the number!

James thought of the double of the square of a number. Then, he added 15 and the result was 65.

Nicola represented the problem: $x^2 + x^2 + 15 = 65$

- Alex used: $2(5)(5) + 15 = 65$
- Ivan proposed: $2(-5)(-5) + 15 = 65$

Explain why the three procedures are correct.

Remember!

Operations hierarchy to solve an arithmetic equation correctly: first, do the operations in parenthesis or brackets.

FIG. 1.1 Operations hierarchy.

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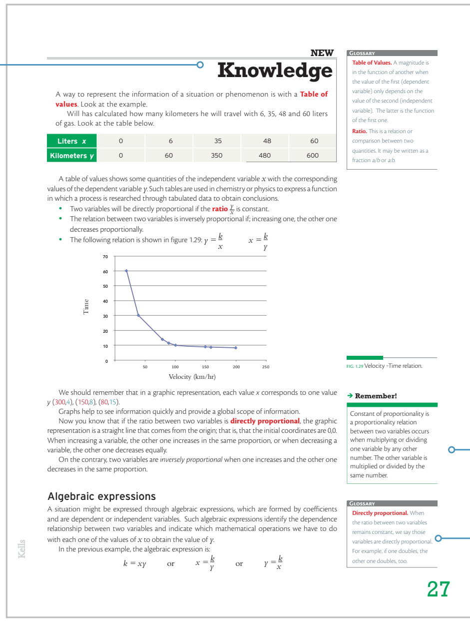
Lesson number and title: It helps to easily find the lessons within the text.

Axis: The mathematical category to improve.

Topic: The specific item to be studied.

Previous Knowledge: It helps you to remember prior knowledge and to understand the new information through questions and activities.



NEW **Knowledge**

A way to represent the information of a situation or phenomenon is with a **Table of values**. Look at the example.

Will has calculated how many kilometers he will travel with 6, 35, 48 and 60 liters of gas. Look at the table below.

Liters x	0	6	35	48	60
Kilometers y	0	60	350	480	600

A table of values shows some quantities of the independent variable x with the corresponding values of the dependent variable y. Such tables are used in chemistry or physics to express a function in which a process is researched through tabulated data to obtain conclusions.

- Two variables will be directly proportional if the ratio $\frac{y}{x}$ is constant.
- The relation between two variables is inversely proportional if, increasing one, the other one decreases proportionally.
- The following relation is shown in figure 1.29: $y = \frac{k}{x}$ $x = \frac{k}{y}$

FIG. 1.29 Velocity - Time relation.

We should remember that in a graphic representation, each value x corresponds to one value y (300,4), (150,8), (80,15).

Graphs help to see information quickly and provide a global scope of information.

Now you know that if the ratio between two variables is **directly proportional**, the graphic representation is a straight line that comes from the origin; that is, that the initial coordinates are 0,0. When increasing a variable, the other one increases in the same proportion, or when decreasing a variable, the other one decreases equally.

On the contrary, two variables are *inversely proportional* when one increases and the other one decreases in the same proportion.

Algebraic expressions

A situation might be expressed through algebraic expressions, which are formed by coefficients and are dependent or independent variables. Such algebraic expressions identify the dependence relationship between two variables and indicate which mathematical operations we have to do with each one of the values of x to obtain the value of y.

In the previous example, the algebraic expression is:

$k = xy$ or $x = \frac{k}{y}$ or $y = \frac{k}{x}$

Glossary: **Table of Values.** A magnitude is in the function of another when the value of the first (dependent variable) only depends on the value of the second (independent variable). The latter is the function of the first one. **Ratio.** This is a relation or comparison between two quantities. It may be written as a fraction a/b or a:b.

→ Remember! Constant of proportionality is a proportionality relation between two variables occurs when multiplying or dividing one variable by any other number. The other variable is multiplied or divided by the same number.

Glossary: **Directly proportional.** When the ratio between two variables remains constant, we say those variables are directly proportional. For example, if one doubles, the other one doubles, too.

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New Knowledge:

The core concepts that you have to use easily.

Remember:

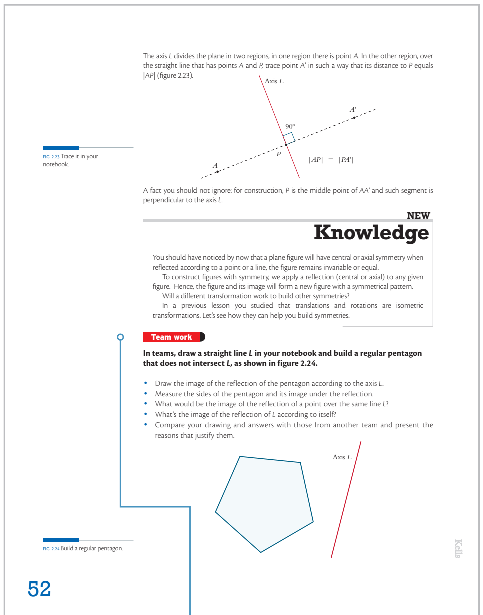
Notes to remind you of other previous content.

Glossary:

It defines vocabulary terms, which you may be unfamiliar with.

Pair work:

The activity to share your understanding with other classmates.



The axis L divides the plane in two regions, in one region there is point A. In the other region, over the straight line that has points A and P, trace point A' in such a way that its distance to P equals |AP| (figure 2.23).

FIG. 2.23 Trace it in your notebook.

A fact you should not ignore: for construction, P is the middle point of AA' and such segment is perpendicular to the axis L.

NEW **Knowledge**

You should have noticed by now that a plane figure will have central or axial symmetry when reflected according to a point or a line, the figure remains invariable or equal.

To construct figures with symmetry, we apply a reflection (central or axial) to any given figure. Hence, the figure and its image will form a new figure with a symmetrical pattern.

Will a different transformation work to build other symmetries?

In a previous lesson you studied that translations and rotations are isometric transformations. Let's see how they can help you build symmetries.

Team work

In teams, draw a straight line L in your notebook and build a regular pentagon that does not intersect L, as shown in figure 2.24.

- Draw the image of the reflection of the pentagon according to the axis L.
- Measure the sides of the pentagon and its image under the reflection.
- What would be the image of the reflection of a point over the same line L?
- What's the image of the reflection of L according to itself?
- Compare your drawing and answers with those from another team and present the reasons that justify them.

FIG. 2.24 Build a regular pentagon.

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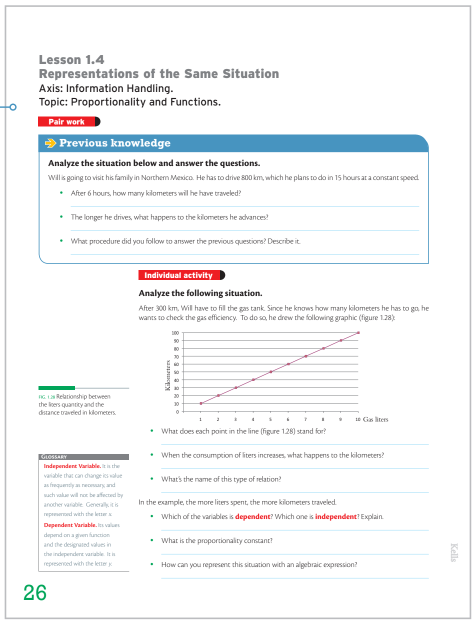

Lesson 1.4
Representations of the Same Situation
Axis: Information Handling.
Topic: Proportionality and Functions.

Pair work

Previous knowledge

Analyze the situation below and answer the questions.

Will is going to visit his family in Northern Mexico. He has to drive 800 km, which he plans to do in 15 hours at a constant speed.

- After 6 hours, how many kilometers will he have traveled?
- The longer he drives, what happens to the kilometers he advances?
- What procedure did you follow to answer the previous questions? Describe it.

Individual activity

Analyze the following situation.

After 300 km, Will have to fill the gas tank. Since he knows how many kilometers he has to go, he wants to check the gas efficiency. To do so, he drew the following graphic (figure 1.28).

FIG. 1.28 Relationship between the liters quantity and the distance traveled in kilometers.

- What does each point in the line (figure 1.28) stand for?
- When the consumption of liters increases, what happens to the kilometers?
- What's the name of this type of relation?

In the example, the more liters spent, the more kilometers traveled.

- Which of the variables is **dependent**? Which one is **independent**? Explain.
- What is the proportionality constant?
- How can you represent this situation with an algebraic expression?

Glossary: **Independent Variable.** It is the variable that can change its value as frequently as necessary, and such value will not be affected by another variable. Generally, it is represented with the letter x. **Dependent Variable.** Its values depend on a given function and the designated values in the independent variable. It is represented with the letter y.

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Team work:

The activity to develop mini projects within the lesson.

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How to use this book

Session information: In this section, you will see the course pacing, week and session. Consider each session is fifty minutes long to cover a 40 week course and you also have the expected learning outcome per session.

Content Delivery: In this section, you will find didactic recommendations to deliver information in class.

Skills Development: This is the list of strategies you will be using in the session.

Evaluation of content: This is the description of how you should evaluate learning outcomes.

SESSION INFORMATION

Week: 1

Session: 2

Expected Learning Outcome: Problem solving using simple quadratic equations and personal procedures or inverse operations.

CONTENT DELIVERY

Start: Draw a square on the board, just like figure 1.2. Ask students to work in pairs to answer the questions on top of the page. Check their work while walking along the classroom. Elicit answers.

Development: Have students read the section *New knowledge*. Ask them to find the area of the square you first drew on the board. Ask them: *What's the formula to calculate the area of the square?* Have a student write it on the board. Ask: *Is this a cubic or quadratic equation? Why is that?* Elicit answers. Ask them to read the riddle and help them analyze it. Step by step, have different volunteers write the reasoning procedure on the board.

Closing: Have students analyze the results in the table and find the correct ones, individually. Elicit answers and solve them on the board with the help of different students.

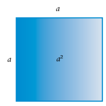

FIG. 1.2 What is a squared number?

Pair work

Analyze the following sentences and answer the questions.

- Look at figure 1.2 and explain what a squared number means.
  It means to multiply a number by itself, as calculating the area of a square.
- Explain what happens when we multiply two negative numbers.
  The result is positive because $(-)(-) = +$
- Which numbers solve the equation? Discuss it with a partner.
  5 and -5.

NEW Knowledge

Quadratic equations represent and solve different problems in real life. To solve equations like these first we ought to use algebraic language and find the appropriate equation.

Mary invited her friends over. She told them that she lives on Independence Street and she said the riddle below to find out her house number:

"The square of a number subtracting the same number equals 132."

- What's the house number?
  12
- Describe the procedure you followed to find out the number.
  Some numbers were squared until the result was a little greater than 32.
- Can the result be negative? Explain your reasons to other pair.
  No, because streets are numbered only with positive figures.
- Share your answers with other classmates and describe the procedures you followed.
- With your teacher's help, decide which methods are correct and why.

Analyze the results in the table below. Find the correct ones.

x	0	1	2	6
$\frac{x^2}{2} = 18$	$\frac{(0)^2}{2} = 18$ $\frac{0}{2} \neq 18$	$\frac{(1)^2}{2} = 18$ $\frac{1}{2} \neq 18$	$\frac{(2)^2}{2} = 18$ $\frac{4}{2} \neq 18$	$\frac{(6)^2}{2} = 18$ $\frac{36}{2} = 18$

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SKILLS DEVELOPMENT

Reasoning skills: Abstracting data, generalizing, making inferences.

Verbal-linguistic skills: Supplying appropriate justifications to a procedure, critiquing the reasoning of others.

Reading skills: Interpreting mathematical information.

EVALUATION OF CONTENT

Check that students at random can find the solutions to the equations.

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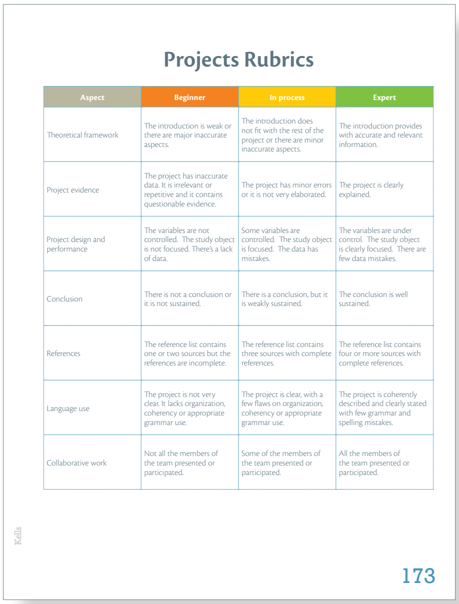
Rubrics: Rubrics to evaluate students' performance to develop projects.



Projects Rubrics

Aspect	Beginner	In process	Expert
Theoretical framework	The introduction is weak or there are major inaccurate aspects.	The introduction does not fit with the rest of the project or there are minor inaccurate aspects.	The introduction provides with accurate and relevant information.
Project evidence	The project has inaccurate data. It is irrelevant or repetitive and it contains questionable evidence.	The project has minor errors or it is not very elaborated.	The project is clearly explained.
Project design and performance	The variables are not controlled. The study object is not focused. There's a lack of data.	Some variables are controlled. The study object is focused. The data has mistakes.	The variables are under control. The study object is clearly focused. There are few data mistakes.
Conclusion	There is not a conclusion or it is not sustained.	There is a conclusion, but it is weakly sustained.	The conclusion is well sustained.
References	The reference list contains one or two sources but the references are incomplete.	The reference list contains three sources with complete references.	The reference list contains four or more sources with complete references.
Language use	The project is not very clear. It lacks organization, coherency or appropriate grammar use.	The project is clear, with a few flaws on organization, coherency or appropriate grammar use.	The project is coherently described and clearly stated with few grammar and spelling mistakes.
Collaborative work	Not all the members of the team presented or participated.	Some of the members of the team presented or participated.	All the members of the team presented or participated.

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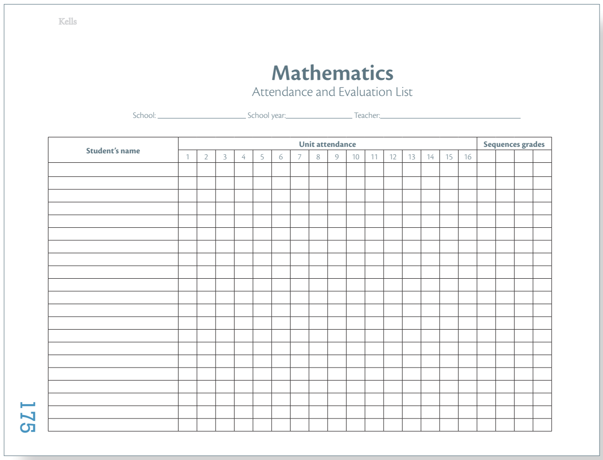
Attendance and Evaluation Grids: These formats will help you keep track of the evaluation procedure specially designed for the content of this book.



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Mathematics

Attendance and Evaluation List

School: ________ School year: ________ Teacher: ________

Student's name	Unit attendance																Sequences grades				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16					

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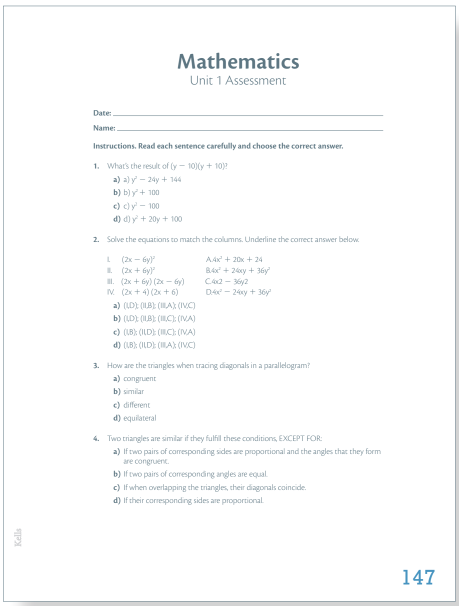
Summative Assessment: Unit evaluation instruments with answer key.



Mathematics

Unit 1 Assessment

Date: ________

Name: ________

Instructions. Read each sentence carefully and choose the correct answer.

1. What's the result of $(y - 10)(y + 10)$?
   a) a) $y^2 - 24y + 144$
   b) b) $y^2 + 100$
   c) c) $y^2 - 100$
   d) d) $y^2 + 20y + 100$
2. Solve the equations to match the columns. Underline the correct answer below.
   I. $(2x - 6y)^2$ — A. $4x^2 + 20x + 24$
   II. $(2x + 6y)^2$ — B. $4x^2 + 24xy + 36y^2$
   III. $(2x + 6y)(2x - 6y)$ — C. 4x2 − 36y2
   IV. $(2x + 4)(2x + 6)$ — D. $4x^2 - 24xy + 36y^2$
   a) (I,D); (II,B); (III,A); (IV,C)
   b) (I,D); (II,B); (III,C); (IV,A)
   c) (I,B); (II,D); (III,C); (IV,A)
   d) (I,B); (II,D); (III,A); (IV,C)
3. How are the triangles when tracing diagonals in a parallelogram?
   a) congruent
   b) similar
   c) different
   d) equilateral
4. Two triangles are similar if they fulfill these conditions, EXCEPT FOR:
   a) If two pairs of corresponding sides are proportional and the angles that they form are congruent.
   b) If two pairs of corresponding angles are equal.
   c) If when overlapping the triangles, their diagonals coincide.
   d) If their corresponding sides are proportional.

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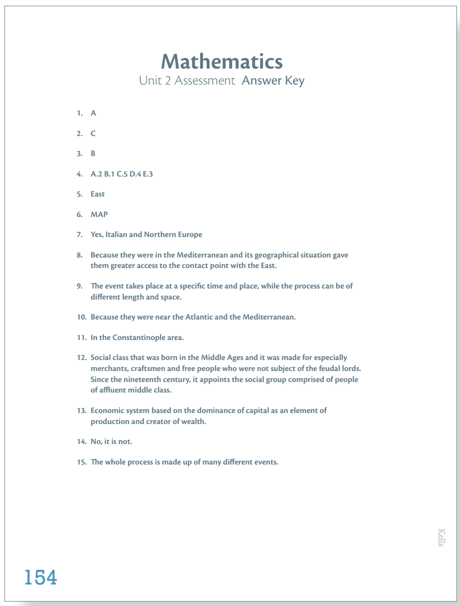

Mathematics

Unit 2 Assessment Answer Key

1. A
2. C
3. B
4. A.2 B.1 C.5 D.4 E.3
5. East
6. MAP
7. Yes, Italian and Northern Europe
8. Because they were in the Mediterranean and its geographical situation gave them greater access to the contact point with the East.
9. The event takes place at a specific time and place, while the process can be of different length and space.
10. Because they were near the Atlantic and the Mediterranean.
11. In the Constantinople area.
12. Social class that was born in the Middle Ages and it was made for especially merchants, craftsmen and free people who were not subject of the feudal lords. Since the nineteenth century, it appoints the social group comprised of people of affluent middle class.
13. Economic system based on the dominance of capital as an element of production and creator of wealth.
14. No, it is not.
15. The whole process is made up of many different events.

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Student book U1

SESSION INFORMATION

Week: 1

Session: 1

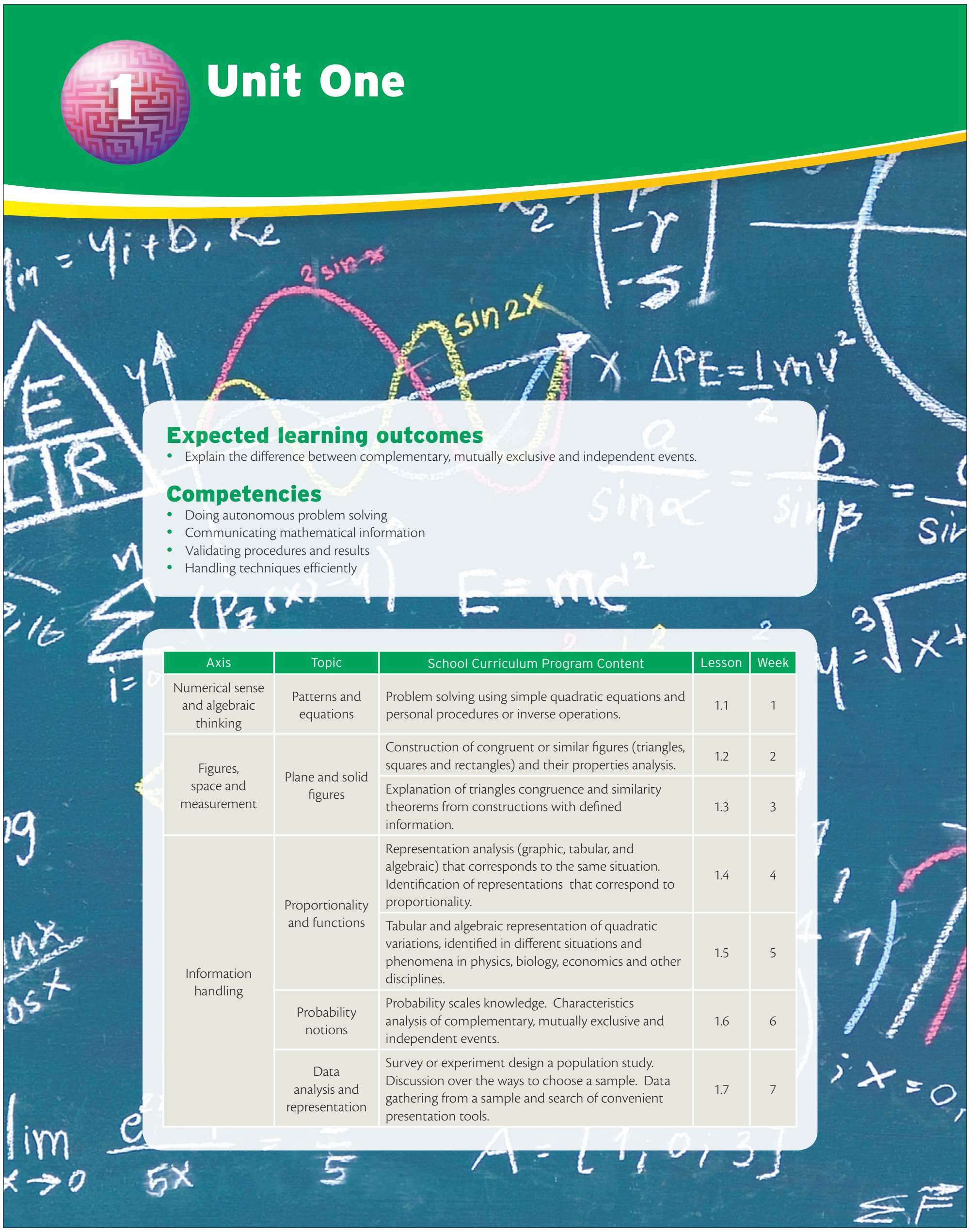
Expected Learning Outcome: Problem solving using simple quadratic equations and personal procedures or inverse operations.

CONTENT DELIVERY

Start: Introduce yourself, the subject, the class schedule, the grading criteria as well as the behavior agreement for peaceful classwork. (For any further information regarding the behavior agreement, look at pages 169 and 170).

Development: Students read the objectives of the unit. Check how familiar they are with the topics by using the diagnostic test that you can find at www.kells-education.co.uk or on pages 145 and 146 in this guide.

Closing: Students identify the topics in the unit they consider will be hard to understand. Then, they will make a studies plan. Ask: *What do you need to do in order to understand those topics? Further practice? Mind maps? A cheating paper with formulas? Mnemonics?* Have students decide on the best way to master such topics individually, and note it down.



1

Unit One

Expected learning outcomes

- Explain the difference between complementary, mutually exclusive and independent events.

Competencies

- Doing autonomous problem solving
- Communicating mathematical information
- Validating procedures and results
- Handling techniques efficiently

Axis	Topic	School Curriculum Program Content	Lesson	Week
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		Explanation of triangles congruence and similarity theorems from constructions with defined information.	1.3	3
Information handling	Proportionality and functions	Representation analysis (graphic, tabular, and algebraic) that corresponds to the same situation. Identification of representations that correspond to proportionality.	1.4	4
		Tabular and algebraic representation of quadratic variations, identified in different situations and phenomena in physics, biology, economics and other disciplines.	1.5	5
	Probability notions	Probability scales knowledge. Characteristics analysis of complementary, mutually exclusive and independent events.	1.6	6
	Data analysis and representation	Survey or experiment design a population study. Discussion over the ways to choose a sample. Data gathering from a sample and search of convenient presentation tools.	1.7	7

SKILLS DEVELOPMENT

Metacognitive skills: Identifying areas of opportunity.

Reading skills: Scanning, skimming, sequencing, reading for detail.

Interpersonal skills: Introducing themselves.

EVALUATION OF CONTENT

Check students' studies plan.

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Lesson 1.1
Problem Solving Using Simple Quadratic Equations

Axis: Numerical Sense and Algebraic Thinking.

Topic: Patterns and Equations.

Pair work

Previous knowledge

Solve the following problems and write down the answers.

- Lucius triples his son's age. In four years time, the sum of their ages will be 88 years. How old is each one now? Why?
  Lucius is 63, his son is 21. 63 is 21 three times, when added we get 84, when adding 4 we get 88.
- Describe step by step the procedure you used to calculate the result. 4 were taken from 88, Lucius is $3x$ if his son is x; therefore, $3x + x = 84$. Then $4x = 84$. Hence, $x = 21$.
- How would you represent the problem using an algebraic expression or a quadratic expression?
  $3x + x + 4 = 88$
- Explain the algebraic expression or quadratic expression that you formulated.
  $3x$ is Lucius' age, x is his son's age, 4 is added and everything equals 88.
- Discuss your answers. Share your answers.
  Students develop their own answers.

Individual activity

Find out the number!

James thought of the double of the square of a number. Then, he added 15 and the result was 65.

Nicola represented the problem: $x^2 + x^2 + 15 = 65$

- Alex used: $2(5)(5) + 15 = 65$
- Ivan proposed: $2(-5)(-5) + 15 = 65$

Explain why the three procedures are correct.

Remember!

Operations hierarchy to solve an arithmetic equation correctly: first, do the operations in parenthesis or brackets.

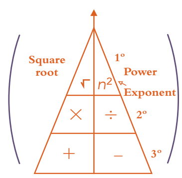

FIG. 1.1 Operations hierarchy.

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SESSION INFORMATION

Week: 1

Session: 1

Expected Learning Outcome: Problem solving using simple quadratic equations and personal procedures or inverse operations.

CONTENT DELIVERY

Start: Have students work with a partner to read the questions in the section *Previous Knowledge*. Once they finish, elicit answers.

Development: Have students read the problem in the *Individual Activity*. Elicit for answers about why the three procedures are correct. Solve each problem step by step, remember that math is hard to do for young people. Have them analyze the operations hierarchy image in order to remember how to solve equations. Write down five to ten more similar examples.

Closing: Check two more examples with them in order to make sure they understand how to solve them.

Homework: Students have to finish the rest of the examples you gave.

SKILLS DEVELOPMENT

Reading skills: Interpreting statements.

Mathematizing skills: Using symbolic, formal and technical language and operations.

Reasoning skills: Generalizing.

EVALUATION OF CONTENT

Check all of the exercises the following session.

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SESSION INFORMATION

Week: 1

Session: 2

Expected Learning Outcome: Problem solving using simple quadratic equations and personal procedures or inverse operations.

CONTENT DELIVERY

Start: Draw a square on the board, just like figure 1.2. Ask students to work in pairs to answer the questions on top of the page. Check their work while walking along the classroom. Elicit answers.

Development: Have students read the section *New knowledge*. Ask them to find the area of the square you first drew on the board. Ask them: *What's the formula to calculate the area of the square?* Have a student write it on the board. Ask: *Is this a cubic or quadratic equation? Why is that?* Elicit answers. Ask them to read the riddle and help them analyze it. Step by step, have different volunteers write the reasoning procedure on the board.

Closing: Have students analyze the results in the table and find the correct ones, individually. Elicit answers and solve them on the board with the help of different students.

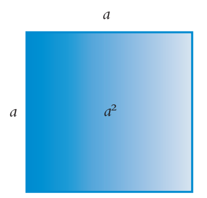


FIG. 1.2 What is a squared number?

Pair work

Analyze the following sentences and answer the questions.

- Look at figure 1.2 and explain what a squared number means.
  It means to multiply a number by itself, as calculating the area of a square.
- Explain what happens when we multiply two negative numbers.
  The result is positive because $(-)(-) = +$
- Which numbers solve the equation? Discuss it with a partner.
  5 and -5.

NEW

Knowledge

Quadratic equations represent and solve different problems in real life. To solve equations like these first we ought to use algebraic language and find the appropriate equation.

Mary invited her friends over. She told them that she lives on Independence Street and she said the riddle below to find out her house number:

"The square of a number subtracting the same number equals 132."

- What's the house number?
  12
- Describe the procedure you followed to find out the number.
  Some numbers were squared until the result was a little greater than 32.
- Can the result be negative? Explain your reasons to other pair.
  No, because streets are numbered only with positive figures.
- Share your answers with other classmates and describe the procedures you followed.
- With your teacher's help, decide which methods are correct and why.

Analyze the results in the table below. Find the correct ones.

x	0	1	2	6
$\frac{x^2}{2} = 18$	$\frac{(0)^2}{2} = 18$ $\frac{0}{2} \neq 18$	$\frac{(1)^2}{2} = 18$ $\frac{1}{2} \neq 18$	$\frac{(2)^2}{2} = 18$ $\frac{4}{2} \neq 18$	$\frac{(6)^2}{2} = 18$ $\frac{36}{2} = 18$

Kells

SKILLS DEVELOPMENT

Reasoning skills: Abstracting data, generalizing, making inferences.

Verbal-linguistic skills: Supplying appropriate justifications to a procedure, critiquing the reasoning of others.

Reading skills: Interpreting mathematical information.

EVALUATION OF CONTENT

Check that students at random can find the solutions to the equations.

Kells

Check table 1.1 and complement table 1.2. Once you have finished, compare your work with a partner and discuss the differences you find.

Verbal Language	Algebraic Language
The square of a number minus the same number equals 90.	$x^2 - x = 90$
Half the sum of the triple of a number and add the quintuple of a number.	$\frac{3x + 5y}{2}$

Table 1.1

Verbal Language	Algebraic Language
	$x^2 + 10 = x + 30$
Half the sum of two squares.	
	$x(2x + 10) = 48$
Find the third of a square root of a number.	

Table 1.2

Individual activity

Let's work! Analyze the following situation.

Half the square of a number is 18.
Which number is that?
The first thing to do is to change verbal language into algebraic language and then, formulate the equation.
Which of the following equations represents the situation?

$$\frac{x}{2} = 18 \qquad \frac{x^2}{2} = 18 \qquad \frac{x}{2^2} = 18$$

Using a square root to solve equations

The extraction of roots is the inverse operation of exponentiation.
When $x^2 = n$, where n is any real number, then $x = \pm\sqrt{n}$. All positive numbers greater than 0 have two square roots and can be:
a) Exact square roots
A root is exact when we find a square elevated number to be exactly the same radicand, without remainder. Look at figure 1.3.

Radical → $\sqrt{16} = 4$ ← Root or base; Radicand → 16

$16 = 4^2$ ← Power

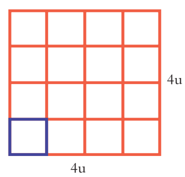


FIG. 1.3 The radicand is a perfect square.

b) Non entire square roots
A square root is not entire when the radicand is not a perfect square.
Square root of 56:

$$+\sqrt{56} = 7.48 \text{ and } -\sqrt{56} = -7.48$$

When to use the property of a square root? To simplify the result, we may write it: ±7.48 It is used to solve equations in the following way:

$$x^2 - 100 = 0$$

Kells

SESSION INFORMATION

Week: 1

Session: 3

Expected Learning Outcome: Problem solving using simple quadratic equations and personal procedures or inverse operations.

CONTENT DELIVERY

Start: Write on the board three quadratic equations. Ask students to write with words how the equations are read. Elicit answers in total class.

Development: Have students read table 1.1 and analyze it along with them. Tell students to complete table 1.2 individually. Ask for the answers at random and elicit answers. Then, guide students asking questions on how to find the appropriate equation in the *Individual activity*. Ask a student to write the square root of 16 in mathematical language. Step by step, explain the parts of exact square roots. Explain why figure 1.3 illustrates the square root of 16, which is exact. Have students do at least 10 more examples, prior to moving on to non-entire square roots.

Closing: Have students do at least 10 examples of non-entire square roots so that they can clearly see the differences between entire and non-entire square roots.

SKILLS DEVELOPMENT

Reasoning skills: Discovering relations.

Mathematical skills: Using formal operations, attending to precision.

EVALUATION OF CONTENT

Students should be able to solve exact and non exact square roots correctly.

Kells

SESSION INFORMATION

Week: 1

Session: 4

Expected Learning Outcome: Problem solving using simple quadratic equations and personal procedures or inverse operations.

CONTENT DELIVERY

Start: Write two examples of exact square roots on the board. Solve them out as a total class, checking step by step that they can follow the procedure.

Development: Have students read the example on top of the page. Give students a short time to read it, analyze it along with them, and have a volunteer solve it doing the operations on the board. Have at least other five examples done on the board by students. Guide them by writing down any other piece of information they need to successfully solve the square root.

Closing: Have students do the square root algorithm by reading bullet by bullet and doing it on the board, to check exactly what to do and how to do it. Clarify any questions that students might have.

Homework: Prepare 10 to 20 similar exercises and have different students dictate them to the class and solve them for homework.

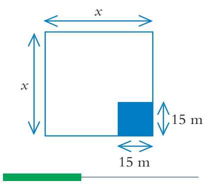


FIG. 1.4 How much does each side of the land measure?

Example:
The size of a piece of land where an exhibit will be organized is 3 619 m². A part of the land is 15 m × 15 m and it is used as a conference hall. Look at figure 1.4.
How long is each side of the land?
The steps to solve the problem are:
Write the quadratic equation. Remember to translate the verbal language to algebraic language.

$$x^2 - 225 = 3\,619$$

Then, isolate the variable:

$$x^2 - 225 + 225 = 3\,619 + 225$$

$$x^2 = 3\,844$$

To get x calculate $\pm\sqrt{3\,844}$ ← square root property

The problem has two solutions, because: (62) (62) = 3 844 and (−62)(−62) = 3 844. Therefore,

$$x_1 = 62 \qquad x_2 = -62$$

- Is the square root in the previous problem positive or negative?
- Thus, how many solutions does the problem have? Explain your answers.

Individual activity

Methods and techniques

Let's follow the square root algorithm with the following example.

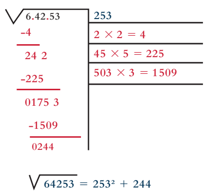


FIG. 1.5 Square root algorithm.

- From right to left, the quantity is divided in periods of two (figure 1.5).
- Use a number that multiplied by itself gives 6 or the closest to it. In this case, the number is 2, because 2 × 2 = 4.
- We take away 6 − 4 = 2 and next to this number we take down the following period, 42 in this case. Therefore, the number becomes 242.
- The 2 is doubled and it becomes 4 (second auxiliary line) and again the 4 is written on the third auxiliary line.
- We calculate how many times 4 is used in 24 (absolute value). In this case it's 6, but since 4 is there, it becomes 46 × 6 = 276 and 276 is greater than 242. Then a smaller number is given to 4, in this case 5 forming 45 and we multiply it by 5. The multiplication has to give a number equal to or smaller than 242. In this case, the multiplication is 225. From 242 take away 225, and it gives 17.
- Next to 17 we take down the following period, 53 in this case, forming 1 753. The 25 of the root is doubled, giving 50 and a number that multiplied by 50 gives a number close to 175, which in this case is 50 × 3 = 150.
- We add 3 to the 503 × 3 = 1509.
- From 1 753 take away 1 509, and it gives 244.
- To check the result, we multiply the square root and add the remainder.

Kells

SKILLS DEVELOPMENT

Mathematizing skills: Using symbolic language and operations, manipulating operations, attending to precission.

EVALUATION OF CONTENT

Check the procedures they follow and the results they get.

Kells

Exercises and application

I. Solve the following problems and complete the tables individually.

1. The square of a number plus ten is 410. What's the number?

Equation	Isolate the variable	Result
$x^2 + 10 = 410$	$x = \sqrt{410 - 10}$	$x = 20$

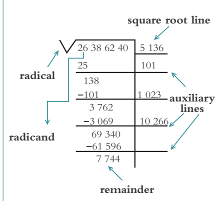
2. The square of a number plus the triple of the same number is 54. What's the number?

Equation	Isolate the variable	Result
$x^2 + 3x = 54$		$x = 6$ or $x = -9$

3. The product of two consecutive numbers is 1 640. What numbers are these?

Equation	Isolate the variable	Result
$x(x + 1) = 1640$	Students	

→ Remember!



Students might use a number of methods to isolate the variable.

$x = 40 \therefore \triangleright 40$ and 41

II. In figures (1.6) and (1.7) find and write the value of x in the corresponding place. Answer the questions.

4. The area of the rectangle in figure 1.6 is 285 u^2.
   - What value does x have?
     15
   - How do you know that?
     Because the square root of 285 is 16.88. If $16 \times 20 = 320$, I multiplied $15 \times 19 = 285$
   - What's the equation that represents this situation?
     $x(x + 4) = 285$

5. The area of the rectangle in figure 1.7 is 506 u^2.
   - What value does x have?
     20
   - How do you know that?
     Because $\sqrt{506} = 22.49$. 22 or 23 equal 506.
   - What's the equation that represents this situation?
     $(x + 2)(x + 3) = 506$
   - Can the result be negative? explain your answer.
     No, because there are not negative values in areas.

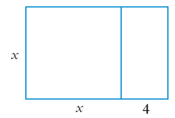


FIG. 1.6 What's the value of x?

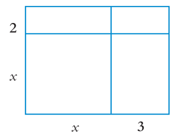


FIG. 1.7 What's the value of x?

III. Find the values of x in the equations below.

a) $x^2 - 9 = 0$
b) $(x - 10)^2 = 400$

Kells

SESSION INFORMATION

Week: 1

Session: 5

Expected Learning Outcome: Problem solving using simple quadratic equations and personal procedures or inverse operations.

CONTENT DELIVERY

Start: Check three of the homework exercises in order to check the procedure. Have students write the procedure to follow on the board. It's important to remember that students do need lots of reinforcement in order to correctly use mathematical procedures.

Development: Give students 5 minutes to answer part 1. Check answers one by one on the board. Ask students at random for the answers. Draw the figures 1.6 and 1.7 on the board. Help them answer the questions by guiding them on how to do it on the board. Make sure everyone follows you. Follow the procedure step by step.

Closing: Take a soft, small ball with you. Students will pass it on while telling multiples of 7. If somebody makes a mistake, he will be the secretary to solve the equations on the board.

Homework: Ask students to take a protractor and set of squares the following class.

SKILLS DEVELOPMENT

Mathematizing skills: Interpreting mathematical information in relation to the situation, manipulating symbolic expressions.

Strategic skills: Selecting and implementing strategies.

EVALUATION OF CONTENT

Check students' answers to the activities. Make sure they all have answered the exercises.

Kells

SESSION INFORMATION

Week: 2

Session: 6

Expected Learning Outcome: Construction of congruent or similar figures (triangles, squares and rectangles) and their properties analysis.

CONTENT DELIVERY

Start: Have students work individually in order to answer the questions on page 16. Give them a few minutes while you walk around the classroom. Once three people have finished, give the rest two more minutes to finish.

Development: Use a large dice in order to choose the students to answer the questions (different students will have to do so). Discuss as a class on the answers, help students draw conclusions and if necessary, students will have to restate their answers to the questions.

Closing: Ask students to tell in their own words the meaning of congruence and similarity. Then, ask students to draw in their notebooks two congruent triangles and two similar triangles using the protractor and set squares they were told to take to class.

Lesson 1.2
Congruent or Similar Figures; Properties Analysis

Axis: Figures, Space and Measurements.

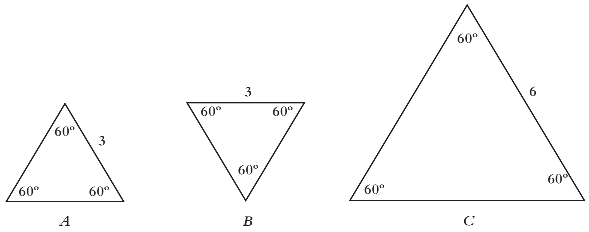
Topic: Plane and Solid Figures.

Individual activity

Previous knowledge

Look at the following triangles (figure 1.8) and answer the questions.

FIG. 1.8 Look at the triangles carefully.



Compare triangles *A* and *B*. What's similar? Can you say they are equal? Why?

A and B are equal since their angles are equal and their sides have exactly the same measurements.

Now, compare triangle *C* to triangles *A* and *B*. What do you see? Are the three triangles equal? Explain why.

The three triangles have equal angles but C has larger sides. C is just similar to A and B.

Two triangles are *congruent* if they are equal; that is, if the sides and angles of one triangle are equal to those in the other triangle.

Triangles *A* and *B* are congruent because they comply with the previous conditions.

But triangle *C* is just *similar* to the other two, because its corresponding internal angles have the same measurements and its sides are NOT equal, but proportional. This is the difference between congruence and similarity.

How can you know when two triangles are congruent and when they are not?

Students develop their own hypothesis based upon the upper triangles analysis.

Kells

SKILLS DEVELOPMENT

Verbal-linguistic skills: Presenting procedures, supplying appropriate justifications to a procedure, critiquing the reasoning of others.

Reasoning skills: Discovering relations, making inferences, providing and checking a justification, making conjectures.

EVALUATION OF CONTENT

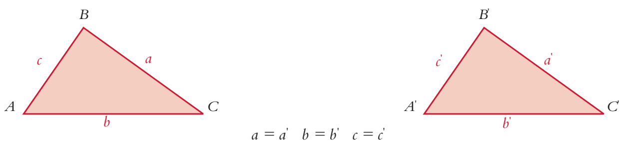
Ask students at random to justify why their triangles are congruent or similar.

Kells

There are three universal rules to check the congruence of two triangles, which we denote:
Rule One:

Side-Side-Side or SSS theorem

(figure 1.9)



$a = a' \quad b = b' \quad c = c'$

FIG. 1.9 SSS theorem.

Rule Two:

Side-Angle-Side or SAS theorem

(figure 1.10)

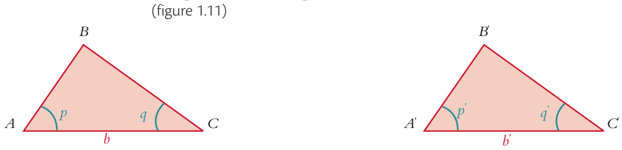
$b = b' \quad p = p' \quad c = c'$

FIG. 1.10 SAS theorem.

Rule Three:

Angle-Side-Angle or ASA theorem

(figure 1.11)



$q = q' \quad b = b' \quad p = p'$

FIG. 1.11 ASA theorem.

Be careful! There is no AAA theorem, because we can have triangles with the same internal angle measurements, but their corresponding sides can be different as shown in the comparison between ΔADE and ΔABC. Look at figure 1.12.

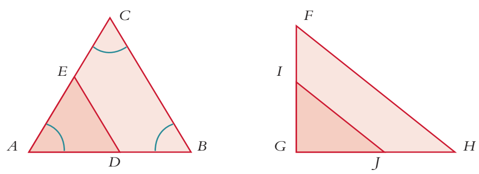


FIG. 1.12 There are triangles with the same measurements of their internal angles, but their corresponding sides are different.

Kells

SESSION INFORMATION

Week: 2

Sessions: 7 - 8

Expected Learning Outcome: Construction of congruent or similar figures (triangles, squares and rectangles) and their properties analysis.

CONTENT DELIVERY

Start: Draw three triangles like the ones on page 16. Ask students to name the triangles that are congruent and the one that is just similar. Ask students to justify their answers.

Development: Have students look at figure 1.9. Ask students: *Are sides A, B, and C equal to sides A', B' and C'? How do you know that? Why is it called SSS theorem?* Have students look at figure 1.10. Ask students: *In this case, what is equal? Why is it called SAS theorem?* Students now look at figure 1.11. Ask students: *Why is it called ASA theorem?* Guide students to name the reasons why it is called the ASA theorem.

Closing: Using the triangles you drew at the beginning of the lesson, have students analyze the theorems application in those triangles on the board. On session 8, have students present the theorems, justify each one and practice till they can tell the reasons why they have such names.

SKILLS DEVELOPMENT

Reasoning skills: Making conjectures, abstracting data, generalizing, discovering relations.

Mathematizing skills: Using symbolic language, understanding symbolic expressions, manipulating symbolic expressions.

EVALUATION OF CONTENT

Students should be able to identify each theorem's application correctly.

Kells

SESSION INFORMATION

Week: 2

Session: 9

Expected Learning Outcome: Construction of congruent or similar figures (triangles, squares and rectangles) and their properties analysis.

CONTENT DELIVERY

Start: Write SSS, SAS, ASA on the board. Have students at random explain each theorem and write the justification of each one on the board.

Development: Have students read the instructions for the *Individual activity*. Ask a student to explain in his own words what they have to do. Then, give them a few minutes to complete the table in activity 2. Copy the table on the board and have different students complete it. Discuss the answers in total class.

Closing: Have a student read the instructions in the *Teamwork* activity. Have a different student explain in his own words what they have to do. Organize teams and give them two minutes to discuss. Check answers in total class. Have students finish the exercise on their own. Check answers in total class, discuss the answers if necessary and students will have to restate their answers if it were the case.

Individual activity

Use your notebook to calculate the following.

1. Consider the following rectangle with vertices *ABCD* and identify the total number of triangles are formed. Write them using their vertices to identify them (figure 1.13).

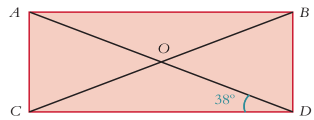


FIG. 1.13 How many triangles are formed?

2. When you have identified the triangles in the previous figure, complete the table:

Triangle	Congruent triangle	Justification	Criterion
$\triangle ACD$	$\triangle ABD$	$\overline{AB} = \overline{CD}\ \measuredangle D = \measuredangle A, \overline{AD} = \overline{DA}$	SAS
$\triangle ACB$	$\triangle BCD$	$\overline{AC} = \overline{BD}\ \measuredangle A = \measuredangle D, \overline{AB} = \overline{CD}$	SAS
$\triangle AOC$	$\triangle BDO$	$\overline{AB} = \overline{BD} \measuredangle 0 = \measuredangle 0, \overline{A0} = \overline{B0}$	SAS
$\triangle COD$	$\triangle ABO$	$\measuredangle A = \measuredangle C, \overline{AB} = \overline{CD}, \measuredangle B = \measuredangle D$	ASA
$\triangle BCD$	$\triangle ABC$	$\overline{AB} = \overline{CD}, \overline{AC} = \overline{BD}, \overline{BC} = \overline{CB}$	SSS

➔ Remember!

If ΔABC is any triangle, the sum of its interior angles is 180°.

Team work

Check the following quadrilaterals (figure 1.14). Decide which one is congruent and explain your reasons.

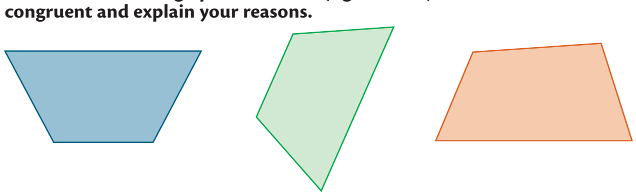


FIG. 1.14 Look at the quadrilaterals and decide which ones are congruent.

- Discuss which quadrilaterals are congruent. You might want to use a graduated ruler and compass to reach your conclusions.
- Consider the following components in figure 1.15 and answer the questions.

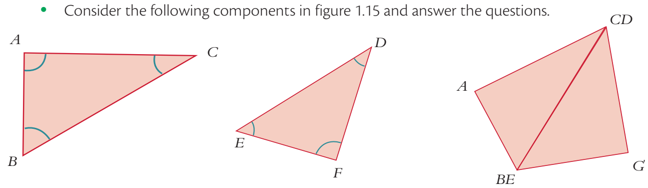


FIG. 1.15 Consider the components of the figures.

- If the addition of the internal angles in a triangle equals 180°, what can you say about the addition of the internal angles of a convex triangle? The sum of its internal angles is 360°.
- If we trace the diagonal *BC*, how much do the six angles measure and how much do they sum? $\measuredangle BAC = 90°$, $\measuredangle ABC = 60°$, $\measuredangle ABD = 30°$, $\measuredangle DEG = 50°$, $EDG = 40°$, $EGD = 90°$
- Does that condition apply to all triangles? Yes, all convex quadrilateral shapes have internal angles whose sum is 360°.

Kells

SKILLS DEVELOPMENT

Reasoning skills: Discovering relations, providing a justification, checking a justification.

Mathematical skills: Using symbolic language.

EVALUATION OF CONTENT

Students should be able to identify the theorems in different applications or examples.

Kells

Exercises and application

Individually, answer the following exercises. When you have finished, compare your answers with a partner.

1. Look at the triangles in figure 1.16, determine which are congruent and which theorem justifies your answer.

The three triangles are congruent because of the ASA theorem.

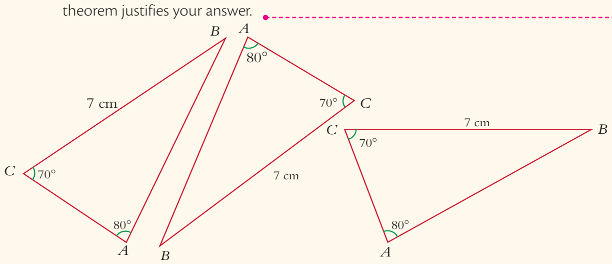


FIG. 1.16 Which triangles are congruent according to which theorem?

2. To demonstrate that the quadrilateral in figure 1.17 $\triangle ABC \equiv \triangle BCD$, means, that the triangles formed are congruent, a student determined that $AC \equiv DC$ and that the angle $CAB \equiv$ angle BCD because they are right-angled triangles. What theorem was used to demonstrate this?

   SAS theorem.

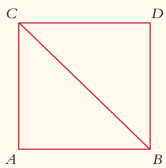


FIG. 1.17 Which theorem justifies congruent triangles?

3. In figure 1.18, $\triangle CDE$ is isosceles. C is half way between AD and D is half way between CB. Which criterion demonstrates that $\triangle ACE \equiv \triangle BDE$?

   SSS

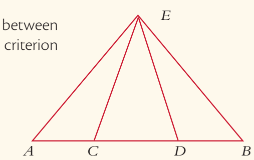


FIG. 1.18 Determine the theorem on congruence.

4. Which of the following sentences are true and which ones are not? Explain your answers.
   - Two right-angled triangles are congruent if their respective acute angles are congruent.
     False. Even though the angles are congruent, the measurements of their sides might not be congruent.
   - Two triangles are congruent if their homologue sides have equal measurements.
     True. When homologue sides have equal measurements, homologue angles have equal measurements.
   - Two triangles are congruent if their respective angles are equal.
     False. Even if the sides are congruent, the measurements of their sides might not be so.
   - To demonstrate that two triangles are congruent you might use the theorem AAS.
     True. Once it is recognized that two sides and their angle is equal, it is known that the rest is equal too.
   - All equilateral triangles are congruent. Why is that?
     False. They have to comply with the criteria ASA, SAS, or SSS.

Kells

SESSION INFORMATION

Week: 2

Session: 10

Expected Learning Outcome: Construction of congruent or similar figures (triangles, squares and rectangles) and their properties analysis.

CONTENT DELIVERY

Start: Have students look at the triangles in exercise 1. Ask students: *Are they similar? Are they congruent? Why is that?* Discuss the answers in total class. Write the theorems on the board, as students give their reasons on why the triangles are congruent.

Development: Have students do exercises 2, and 3. Remember to segment the exercises check. Once they finish one exercise, go through the answers along with the class, and make any necessary clarifications they need.

Closing: Students are to determine which sentences are true or false and explain the reasons why they think so. Give them a few minutes to analyze and elaborate the justification. Once they finish, check answers in total class.

Homework: Students take to class a protractor and set square the following session.

SKILLS DEVELOPMENT

Reasoning skills: Making inferences, making conjectures, providing and checking a justification.

Verbal-linguistic skills: Supplying appropriate justifications, critiquing the reasoning of others.

Reading skills: Interpreting mathematical information.

EVALUATION OF CONTENT

Check that students can name the theorems and the justification on why the triangles are congruent or not.

Kells

SESSION INFORMATION

Week: 3

Session: 11

Expected Learning Outcome: Explanation of triangles congruence and similarity theorems from constructions with defined information.

CONTENT DELIVERY

Start: Ask students to read the instructions of the *Individual activity*. Ask for a student at random what they have to do. Go through the answers along with the class. Critique their reasoning so that they can easily draw conclusions , remember when critiquing it is important to go step by step on the procedure so as to clarify any possible mistake or turning point.

Development: Have students read the second part of the page. Help them analyze sentence by sentence with triangles on the board.

Closing: Students will draw other different triangles in their notebooks. Have students do a similar analysis.

Lesson 1.3
Triangle Congruence and Similarity Theorems

Axis: Figures, Space and Measurements.
Topic: Plane and Solid Figures.

Individual activity

Previous knowledge

Let's remember the properties of the addition of internal angles of a triangle and the properties of the angles between parallels.

Measure or calculate? Consider the following triangles (figure 1.19).

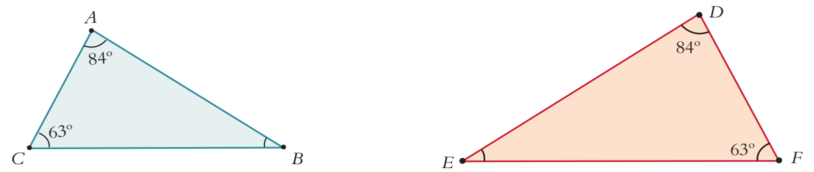


FIG. 1.19 Gauge the internal angles of the triangles.

Now that you know the measurements of the interior angles of the triangles, even considering any small margin of error, calculate:

- How compare the angle $\measuredangle CBA$ according to the angle $\measuredangle DEF$?
- Are they the same or different?
- How much is the sum of the interior angles in a triangle?

Then, for the previous triangles, we will have:

$\measuredangle CBA + \measuredangle BAC + \measuredangle ACB = 180°$
$\measuredangle DEF + \measuredangle FDE + \measuredangle EFD = 180°$
Which equals:
$\measuredangle CBA + 84° + 63° = 180°$
$\measuredangle DEF + 84° + 63° = 180°$
Isolate the variable as you learned in previous courses.
When simplifying we get:
$\measuredangle CBA = 33°$ and $\measuredangle DEF = 33°$

This reasoning, applied to any pair of triangles ΔABC and ΔDEF, will show that the angle $\measuredangle CBA$ will equal the angle $\measuredangle DEF$, as long as each one of the rest of the angles in ΔABC equals one in ΔDEF (figure 1.20).

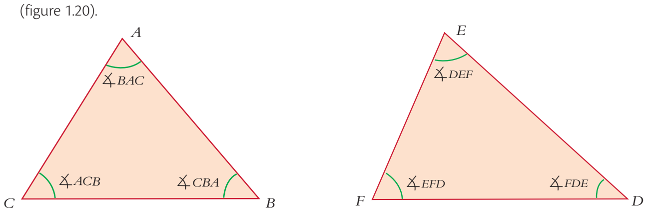


FIG. 1.20 Look at the triangles carefully.

Kells

SKILLS DEVELOPMENT

Reasoning skills: Discovering relations, making conjectures, providing and checking a justification, generalizing.

Mathematical skills: Using symbolic expressions, using constructs based on definitions; attending to precision.

EVALUATION OF CONTENT

Check students can easily identify when angles are similar.

Kells

Two triangles are similar if the internal angles in one of them are equal to the angles in the other. When solving our example and generalizing it, we get our first theorem on similarity:

AA theorem: Two triangles are similar if two internal angles of one are equal to the corresponding angles of the other.

Look at the example. Consider the triangles ΔLMN and ΔPQR in figure 1.21.

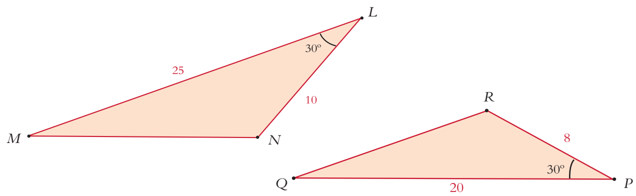


FIG. 1.21 Are the triangles similar?

Are they similar? How can we know that?

- A way to know that would be to measure the missing information in each one and check if the triangles comply with the definition of similarity.
  Two triangles are similar if their corresponding sides are proportional.
- We could also measure one corresponding angle in each one of them, for instance $\measuredangle PRQ$ and $\measuredangle MNL$, check if they are equal and apply the theorem AA.
- Do the two exercises in your notebook; explain why the two triangles are similar or not.

Individual activity

Look at the triangles in figure 1.21 (previous activity) again carefully.

- On a piece of paper, draw both triangles with the same features.
- Overlap the two triangles as shown in figure 1.22.

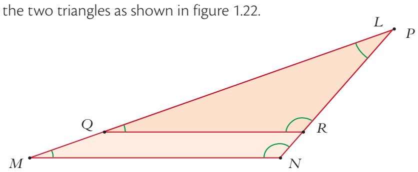


FIG. 1.22 They are the same triangles from the previous activity, but overlapped.

- According to your drawing, are segments QR and MN parallel?
- Explain your answer to other students in your group and listen to their arguments.

NEW Knowledge

From the lengths:	We can calculate the quotients:	These calculations show that segments *LM, LN* and *PQ, PR* are proportional. That is:
$LM = 25$ $LN = 10$ $PQ = 20$ $PR = 8$	$\frac{LM}{PQ} = \frac{25}{20} = 1.25$ $\frac{LN}{PR} = \frac{10}{8} = 1.25$	$\frac{LM}{PQ} = \frac{LN}{PR}$

Kells

SESSION INFORMATION

Week: 3

Session: 12

Expected Learning Outcome: Explanation of triangles congruence and similarity theorems from constructions with defined information.

CONTENT DELIVERY

Start: Copy the triangles on page 21 on the board. Ask students if they are congruent or similar and why. Have students write their reasoning on the board.

Development: Tell students to read the AA theorem, look at the triangles and say if they are similar and why. Have students practice with at least five similar examples in which they see the AA theorem applied or not.

Closing: Have students read the *New knowledge* table. Ask students to prepare a class in which they explain that table. Select at random a student to present it. Encourage other students to critique his reasoning using the information in the book and going step by step on their solution so as to clarify problems or possible imprecisions.

SKILLS DEVELOPMENT

Mathematical skills: Manipulating symbolic expressions, using constructs based on formal systems, modeling.

Reasoning skills: Generalizing, reasoning quantitatively.

EVALUATION OF CONTENT

Students should be able to explain the table in the section *New knowledge* under your guidance.

Kells

SESSION INFORMATION

Week: 3

Session: 12

Expected Learning Outcome: Explanation of triangles congruence and similarity theorems from constructions with defined information.

CONTENT DELIVERY

Start: Draw figure 1.22 on the board. Different students identify each angle and segment on the triangles.

Development: Students read sentence by sentence and analyze the triangles on the board. Guide them, but asking questions. For example: *Are triangles LMN and PQR similar?* Explain your reasons to saying so.

Closing: Guide them to complete the chart by solving in total class step 1, ask them questions giving two options; they will only need to discard one of the options you give. Have students analyze at least other four pairs of triangles in order to determine whether they are similar or not. (The last part of the analysis is on top of page 23).

Let's remember the definition of similar triangles: *Two triangles are similar if their corresponding sides are proportional.*

This indicates that the triangles ΔLMN and ΔPQR in figure 1.21 are similar if:

$$\frac{LM}{PQ}=\frac{LN}{PR}=\frac{MN}{QR}$$

Then, we know that in figure 1.21

$$\frac{LM}{PQ}=\frac{LN}{PR}$$

This indicates that segments *LM, LN* and *PQ, PR* are proportional.

Now, let's use the *Reciprocal of the Theorem of Tales*: If a line s cuts the triangle ΔPMN at points *Q* and *R* in such a way that segments *PQ, QM* and *PR, RN* are proportional, that is $\frac{PQ}{QM}=\frac{PR}{RN}$; hence, the line is parallel to segment *MN*.

In the example in figure 1.22 we know that:

$$\frac{PQ}{PM}=\frac{PR}{PN}$$

On the other hand, we have (figure 1.23).

$$PM = PQ + QM$$
$$PN = PR + RN$$

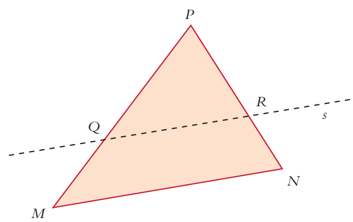


FIG. 1.23 *Reciprocal of the Theorem of Tales.*

If we substitute the first two equalities, we find:

$$\frac{PQ}{PQ+QM}=\frac{PR}{PR+RN}$$

From this last equality, (by simple isolation), we get the following:

$$\frac{PQ+QM}{PQ}=\frac{PR+RN}{PR}$$

And now, in four steps, we will see how we can get the equality of the *Reciprocal of the Theorem of Tales* through the previous equality.

Step 1	Step 2	Step 3	Step 4
$\frac{PQ}{PQ}+\frac{QM}{PQ}=\frac{PR}{PR}+\frac{RN}{PR}$	$1+\frac{QM}{PQ}=1+\frac{RN}{PR}$	$\frac{QM}{PQ}=\frac{RN}{PR}$	$\frac{PQ}{QM}=\frac{PR}{RN}$

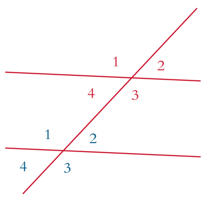


FIG. 1.24 Parallels cut by a transversal line.

We might conclude that segments *MN* and *QR* in our example are parallels.

This is highly convenient if we apply the properties of angles in parallels: *If two parallels are cut by a transversal line, then the corresponding angles are congruent (equal)* (figure1.24).

22

Kells

SKILLS DEVELOPMENT

Mathematical skills: Interpreting mathematical information, manipulating symbolic expressions.

Reasoning skills: Discovering relations.

EVALUATION OF CONTENT

Check that students can easily represent Thales' Reciprocal Theorem.

Kells

The numbers represent angles: equal numbers stand for equal angles.

Thus, $\measuredangle PQR = \measuredangle LMN$ and $\measuredangle QRP = \measuredangle MNL$ (figure 1.22 go back to page 21).

And as $\measuredangle RPQ = \measuredangle NLM = 30^\circ$, the internal angles ΔLMN and ΔPQR are equal, that is, both triangles are similar.

Group activity

Consider the example in figure 1.22 (go back to page 21). Then, answer the following questions.

- At the beginning of the problem, we found out that both triangles have an equal angle: $\measuredangle RPQ = \measuredangle NLM$. If their numerical value were different from 30°, would the triangles be different? Explain why.
- In your notebook, draw your own figures and change the name of the vertices. Using the figures you drew in your notebook, summarize the properties that we studied when we solved the previous example.
- Write them on the board and clarify any questions with your teacher.

We have saw in the example that two triangles, with one angle of the same value and two sides of one triangle proportional to two sides of another, are similar. The value of such angle was not relevant to reach our conclusion; it was enough to know that it was the same in both triangles. So, we can reach our second theorem on similarity:

SAS theorem: Two triangles are similar if two sides of one are proportional to two sides of the other and if the angles between them are equal (figure 1.25).

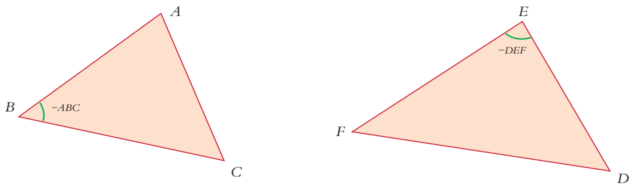


FIG. 1.25 SAS Theorem on congruence.

In other words if: $\frac{AB}{DE} = \frac{BC}{EF}$ and $\measuredangle ABC = \measuredangle DEF$, therefore $\Delta ABC \sim \Delta DEF$.

Let's analyze a similar theorem to the previous one. Consider two triangles ΔABC and ΔDEF with a pair of proportional sides (figure 1.26), demonstrating the equality:

$$\frac{AB}{DF} = \frac{BC}{EF}$$

Kells

SESSION INFORMATION

Week: 3

Session: 13

Expected Learning Outcome: Explanation of triangles congruence and similarity theorems from constructions with defined information.

CONTENT DELIVERY

Start: Write parts of Thales' reciprocal theorem and ask students how to represent it in a triangle. Check answers in total class.

Development: Divide the group in teams and have them read the instructions in the group activity. Ask students to explain what they have to do in their own words. Have students read the bullets and make clear any piece of information you consider is necessary. Check they are on task while walking around the classroom. Check their answers.

Closing: Ask students to find the first theorem on similarity in previous pages (page 21) and copy it in their notebooks, have a student write it on the board. Ask students to represent it. Check answer in total class. Then, ask a student to read the second theorem on similarity. Ask students to read it slowly and explain it to you using figures 1.25 and 1.26.

SKILLS DEVELOPMENT

Reading skills: Interpreting statements.

Mathematical skills: Interpreting mathematical objects, using constructs based on definitions.

Reasoning skills: Generalizing, making inferences.

EVALUATION OF CONTENT

Students prepare a presentation of both theorems on similarity. Choose at random somebody to explain each theorem the following class.

Kells

SESSION INFORMATION

Week: 3

Session: 14

Expected Learning Outcome: Explanation of triangles congruence and similarity theorems from constructions with defined information.

CONTENT DELIVERY

Start: Have different students present the first and second theorems on similarity. Ask students at random questions about the presentations.

Development: Draw two right angle triangles on the board. Ask a student to read the instructions in the *Individual activity*. Read the third theorem on similarity and ask students how they would demonstrate the theorem using the triangles on the board. Guide them to demonstrate it.

Closing: Have students copy the three theorems on similarity and illustrate each one in their notebook.

Homework: Students have to use a tape measure the following class.

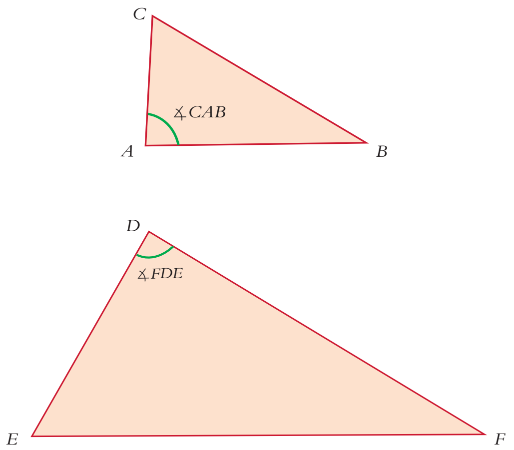


FIG. 1.26 Consider the equality of both triangles.

- What can we say about both triangles?
- Let's suppose that the angle $\measuredangle CAB \geq \measuredangle BCA$ and that $\measuredangle FDE \geq \measuredangle DEF$ (figure 1.26). If in this case $\measuredangle CAB = \measuredangle FDE$, can we say that they are similar?

Individual activity

Copy the triangle ΔA'B'E in the following way:

- Draw the angle A' over the angle A, in such a way that the side $A'E$ corresponds to the side AE (evidently the side $A'B'$ will be on the side AF). Remember that $\measuredangle EA'B' = \measuredangle CAB$, $AB = A'B'$ and $BC = EB'$.
- What happens with segments AC and $A'E$? Are they equal?
- The answer is yes, they are. Hence, the segments BC and EB' are parallel as well. We can now conclude that ΔABC and ΔDEF are similar.

Summarizing, we have gotten our third and last theorem on similarity:

SSA theorem: Two triangles are similar if two sides of one are proportional to two sides of the other and if the major opposite angle of one triangle is equal to the major opposite angle of the other.

In other words, triangles ΔABC and ΔDEF will be similar if sides AB, BC and DF, EF are proportional and if $\measuredangle CAB = \measuredangle FDE$, where $\measuredangle CAB \geq \measuredangle BCA$ and $\measuredangle FDE \geq \measuredangle DEF$.

Kells

SKILLS DEVELOPMENT

Reading skills: Interpreting mathematical information.

Mathematical skills: Using symbolic expressions, using constructs based on definitions, modeling.

Reasoning skills: Checking a justification.

EVALUATION OF CONTENT

Students have to be able to demonstrate the three theorems on similarity.

Kells

Exercises and application

Analyze figure 1.27.

1. How high is the building?
2. Go out to the schoolyard and with the same method calculate how tall the flagpole is.

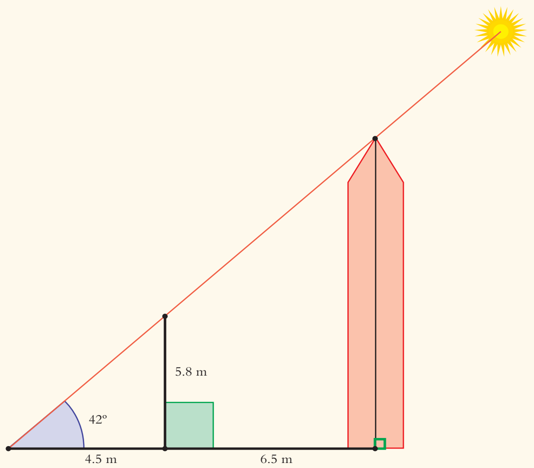


FIG. 1.27 The building projects an 11 m long shadow.

3. Get together with other student in your group and compare your results; describe the procedure you followed in order to get the answer.
4. Backup your answer with reasons and ask for your teacher's help to draw conclusions.

Go out to the schoolyard and calculate with the same method how tall the flagpole or any other pole is. Write down in the space below your procedure and the result.

Answers vary according to the school pole height.

Kells

SESSION INFORMATION

Week: 3

Session: 15

Expected Learning Outcome: Explanation of triangles congruence and similarity theorems from constructions with defined information.

CONTENT DELIVERY

Start: Write the three theorems on similarity on the board. Ask students at random to graphically demonstrate each one. Clarify any question that students might have.

Development: Students analyze the problem then answer the questions. Check their answers on the board in total class.

Closing: Students have to go to the schoolyard in order to calculate how tall the flagpole is. Make sure you establish the rules to leave the classroom, check all of your students are on task and guide their practice.

Homework: Students have to take a map of Mexico with three places they like or would like to visit and the distance between their hometown and those places (for example, Veracruz-Mexico City: 420 km).

SKILLS DEVELOPMENT

Strategic skills: Selecting and implementing strategies.

Mathematical skills: Modeling, transforming a real world problem into a mathematical problem.

EVALUATION OF CONTENT

Check that students can gauge the flagpole using the appropriate method.

Kells

SESSION INFORMATION

Week: 4

Session: 16

Expected Learning Outcome: Representation analysis (graphic, tabular, and algebraic) that corresponds to the same situation. Identification of representations that correspond to proportionality.

CONTENT DELIVERY

Start: Ask students to tell three locations away from their hometown and mark them on a large map of Mexico on the board. Ask different students to read each question in the *Previous knowledge* box and answer each question along with the class. Guide them by using questions with two options.

Development: Individually students will analyze the graph and answer the questions. Give them a few minutes to do so. Check the answers to the questions, one by one, having different students writing the solutions on the board.

Closing: Students have to make a graph with the information of the three places they spotted in the map of Mexico, one in class, the other two for homework; in case most students did not do the homework, assign seven different locations.

Lesson 1.4
Representations of the Same Situation
Axis: Information Handling.
Topic: Proportionality and Functions.

Pair work

Previous knowledge

Analyze the situation below and answer the questions.

Will is going to visit his family in Northern Mexico. He has to drive 800 km, which he plans to do in 15 hours at a constant speed.

- After 6 hours, how many kilometers will he have traveled?
  319.9 km
- The longer he drives, what happens to the kilometers he advances?
  The quantity of kilometers increases.
- What procedure did you follow to answer the previous questions? Describe it.

The velocity is calculated when dividing distance by time, then, the velocity is multiplied by time to know the kilometers to travel.

Individual activity

Analyze the following situation.

After 300 km, Will have to fill the gas tank. Since he knows how many kilometers he has to go, he wants to check the gas efficiency. To do so, he drew the following graphic (figure 1.28):

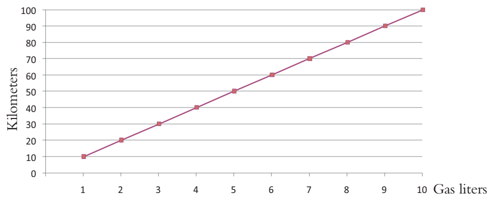


FIG. 1.28 Relationship between the liters quantity and the distance traveled in kilometers.

- What does each point in the line (figure 1.28) stand for?
  The relationship consumed gas vs. traveled kilometers.
- When the consumption of liters increases, what happens to the kilometers?
  The kilometers increase 10 units.
- What's the name of this type of relation?
  Directly proportional relationship.

In the example, the more liters spent, the more kilometers traveled.

- Which of the variables is **dependent**? Which one is **independent**? Explain.
- What is the proportionality constant?
  10.
- How can you represent this situation with an algebraic expression?
  $d = 10L$

The independent variable is the gas liters; the dependent variable is the kilometers because it depends on the gasoline.

GLOSSARY

Independent Variable. It is the variable that can change its value as frequently as necessary, and such value will not be affected by another variable. Generally, it is represented with the letter *x*.

Dependent Variable. Its values depend on a given function and the designated values in the independent variable. It is represented with the letter *y*.

26

Kells

SKILLS DEVELOPMENT

Reading skills: Interpreting mathematical information.

Mathematical skills: Transforming a real world problem into a mathematical problem.

Reasoning skills: Discovering relations.

Strategic skills: Selecting and implementing strategies.

EVALUATION OF CONTENT

Students have to make three graphs using the places and map from their homework papers.

NEW
Knowledge

A way to represent the information of a situation or phenomenon is with a **Table of values**. Look at the example.

Will has calculated how many kilometers he will travel with 6, 35, 48 and 60 liters of gas. Look at the table below.

Liters *x*	0	6	35	48	60
Kilometers *y*	0	60	350	480	600

A table of values shows some quantities of the independent variable x with the corresponding values of the dependent variable y. Such tables are used in chemistry or physics to express a function in which a process is researched through tabulated data to obtain conclusions.

- Two variables will be directly proportional if the **ratio** $\frac{y}{x}$ is constant.
- The relation between two variables is inversely proportional if; increasing one, the other one decreases proportionally.
- The following relation is shown in figure 1.29: $y = \frac{k}{x}$ $\quad x = \frac{k}{y}$

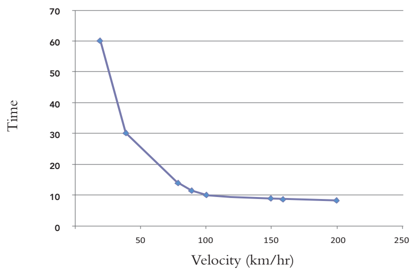


FIG. 1.29 Velocity –Time relation.

We should remember that in a graphic representation, each value x corresponds to one value y (300,4), (150,8), (80,15).

Graphs help to see information quickly and provide a global scope of information.

Now you know that if the ratio between two variables is **directly proportional**, the graphic representation is a straight line that comes from the origin; that is, that the initial coordinates are 0,0. When increasing a variable, the other one increases in the same proportion, or when decreasing a variable, the other one decreases equally.

On the contrary, two variables are *inversely proportional* when one increases and the other one decreases in the same proportion.

Algebraic expressions

A situation might be expressed through algebraic expressions, which are formed by coefficients and are dependent or independent variables. Such algebraic expressions identify the dependence relationship between two variables and indicate which mathematical operations we have to do with each one of the values of x to obtain the value of y.

In the previous example, the algebraic expression is:

$$k = xy \quad \text{or} \quad x = \frac{k}{y} \quad \text{or} \quad y = \frac{k}{x}$$

Glossary

Table of Values. A magnitude is in the function of another when the value of the first (dependent variable) only depends on the value of the second (independent variable). The latter is the function of the first one.

Ratio. This is a relation or comparison between two quantities. It may be written as a fraction *a/b* or *a:b*.

Directly proportional. When the ratio between two variables remains constant, we say those variables are directly proportional. For example, if one doubles, the other one doubles, too.

→ Remember!

Constant of proportionality is a proportionality relation between two variables occurs when multiplying or dividing one variable by any other number. The other variable is multiplied or divided by the same number.

Kells

SESSION INFORMATION

Week: 4

Sessions: 17, 18

Expected Learning Outcome: Representation analysis (graphic, tabular, and algebraic) that corresponds to the same situation. Identification of representations that correspond to proportionality.

CONTENT DELIVERY

Start: Two students will copy their homework graphs on the board, in case nobody has done the homework, have different students develop the distance graph on the board.

Development: Students read the *New knowledge* section, ask students how to build a table of values using the information from the graphs on the board; make pauses, asking them which values will be in the x axis and which ones in the y axis. Ask students to read the definition of directly proportional (write the term on the board) and students have to identify if the table of values they obtained is directly or inversely proportional and why they know that.

Closing: Give students a similar problem and have them build the table of values and graph.

SKILLS DEVELOPMENT

Mathematical skills: Manipulating symbolic expressions, using constructs based on formal systems.

EVALUATION OF CONTENT

Students should be able to create a table of values and graph out of any given problem.

Kells

SESSION INFORMATION

Week: 4

Session: 19

Expected Learning Outcome: Representation analysis (graphic, tabular, and algebraic) that corresponds to the same situation. Identification of representations that correspond to proportionality.

CONTENT DELIVERY

Start: Organize pairs. Have students analyze the table in figure 1.30 and call at random for a student to explain the proportionality between the variables. Help them out by asking two-options questions, for example: *How much does volume decrease between 10 and 5 Pa, 1 cm^3 or 10 cm^3? Now, how much does volume decrease between 10 and 15 Pa, 1 cm^3 or 10 cm^3?*

Development: Step by step help students answer the questions individually in exercise 2, by making two-option questions. If necessary, repeat procedures with different examples.

Closing: Students do the final activity for homework in case you do not have enough time to do it in class. Remember to ask direct questions instead of asking: Do you have any problem? (Students will always be shy to admit they don't get it!)

Exercises and application

1. In pairs, make a table of values with the data in the table (figure 1.30).
   - Explain the proportionality between the variables. Discuss it with another pair.

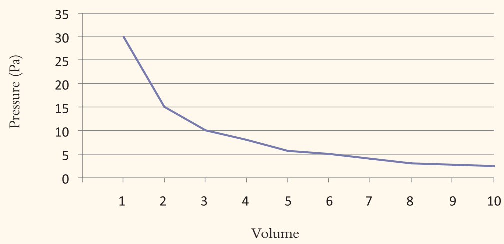


FIG. 1.30 Pressure-Volume graph.

2. With the data in figure 1.30 and the table of values you made, answer the following questions.
   - If the volume increases, what happens to the pressure?
   - If the pressure decreases, what happens to the volume?
   - If pressure is represented by *P*, and volume by *V*, what is the algebraic expression for this situation?
   - What happens to the pressure if the volume triples? Explain your answer.
3. Do the activities using the following situation.
   Mexico is one of the main producers of oil in the world. The algebraic expression that models the quantity of millions of barrels that are produced daily is: $y = 2.5x$
   According to such algebraic expression, complete the following table.

Days	1	2	3	15	180	365
Barrels (in millions)	2.5	5.0	7.5	37.5	450	912.5

   - Explain with words the algebraic expression that was used: $y = 2.5x$.
     It means that 2.5 million barrels are produced per day.
   - Which is the independent variable?
     Days (x).
   - How many barrels are produced in Mexico annually?
     912.5 million barrels.
   - In your notebook make the graph that corresponds to the previous table and indicate the relationship between the two variables. Explain your answers.

4. When studying a gas mass you see that $P = \frac{30}{V}$
   - Build a table of values and calculate the value of P for $V = 0, 1, 2, 3, 5, 6, 10, 15$.
   - What's the algebraic expression for this situation?

Kells

SKILLS DEVELOPMENT

Mathematizing skills: Using constructs based on definitions, manipulating symbolic expressions.

Reasoning skills: Abstracting data, making inferences, making conjectures.

EVALUATION OF CONTENT

Ask for the answers to the questions. If students cannot respond correctly, go back to explain with a similar example and then ask again.

Kells

Lesson 1.5 Representation of Quadratic Variations in a Number of Situations and Disciplines

Axis: Information Handling.
Topic: Proportionality and Functions.

Previous knowledge

Solve the following problem and write down your answers.

Martin needs to buy an 800 m^2 a piece of land. The land is rectangular and its length is twice as much as its width.

- What are the measuresments of the land? Explain how you got the result.
  20×40m. When dividing the piece of land in halves, there are two squares of 400 m^2 and $\sqrt{400} = 20$.
- What's the algebraic expression that represents this situation? Explain it.
  $2x(x) = 800$ or $2x^2 = 800$

Pair work

Analyze the following information and the figures below (figure1.31). Then, answer the questions.

A piece of synthetic grass has a perimeter of 24 m. Louise wants to make the best out of it. Answer the following questions in your notebook.

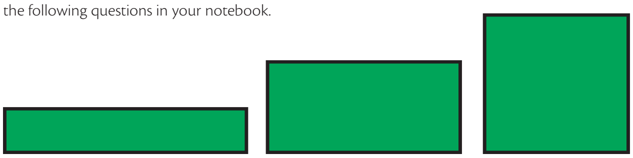


FIG. 1.31 Even though they have the same diameter their areas are different.

- What procedure can you use to calculate the maximum area that can be covered with the synthetic grass?
  Different rectangles might be tested until the one that gauges 6×6.
- Is it absolutely necessary to form a rectangle? Justify your answer.
  No, it might be a square, too.
- If you had to graph the problem, what would the graph be like? What would the two variables to be related be? Justify your answers.
  The variables would be area and the size of a side, the graph would be curved.
- The following proposals might help to solve the problem.
  - Identify the variables.
  - Write how the variables are related.

Kells

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SESSION INFORMATION

Week: 4

Session: 20

Expected Learning Outcome: Representation analysis (graphic, tabular, and algebraic) that corresponds to the same situation. Identification of representations that correspond to proportionality.

CONTENT DELIVERY

Start: Tell students that you have a friend who needs to buy a piece of land of 800 m2 and explain that the piece of land he chose is rectangular; its length is twice as much as its width (draw it on the board). Help them to isolate the variables with the information you have. Help students by making two-option questions so that they can answer both questions.

Development: Draw the three rectangles in the *Pair work* activity on the board. Guide your students through two-option questions in order to help them analyze the situation and therefore, answer the questions correctly.

Closing: Help them find the variables and how the variables are related by asking them, for example: *How many variables are there, two or four?*

SKILLS DEVELOPMENT

Reading skills: Interpreting statements.

Mathematical skills: Interpreting mathematical objects or information in relation to the situation represented, manipulating symbolic expressions.

Reasoning skills: Discovering relations, making inferences.

EVALUATION OF CONTENT

Ask students to find the variables and how the variables are related in other similar situations (five to ten different problems, at least).

Kells

SESSION INFORMATION

Week: 5

Session: 21

Expected Learning Outcome: Tabular and algebraic representation of quadratic variations, identified in different situations and phenomena in physics, biology, economics and other disciplines.

CONTENT DELIVERY

Start: Ask different students to identify the variables in quadrangular shapes, call them to the board, it is preferable to start with the ones who have a better handling of mathematical skills and move forward with the rest of the students until someone who finds math to be specially hard can successfully participate.

Development: Have students read the top of the page. Help them analyze the information step-by-step and eliciting for answers with two-option questions. Solve the first two equations in the table along with your group, little by little, eliciting for the answers in each step you develop. Then, have students complete the table and check it in total class.

Closing: Students read the note in the section *New knowledge*, help them understand what a function is by providing with at least other two examples of functions.

- Pay attention to the fact that a rectangle is mentioned, in which the perimeter is constant. However, we need to know the maximum possible area without changing the value of the first variable.

base × height = ____________ m^2 P = 24 m

- You might help yourselves by tracing geometric representations.
- You might want to make a table to solve the problem. Make one with the base value from 1 to 11. Look at the example and continue with the table:

Base (b)	Height (h)	Perimeter = 24 m	Area = m^2
x	$12 - x$	2( ) + 2( )= 24 m	$(x)(12x-x) = 12x-x^2$
1	11	2(1) + 2(11) = 24 m	11
2	10	2(2) + 2(10) = 24	20
3	9	2(3) + 2(9) = 24	27
4	8	2(4) + 2(8) = 24	32
5	7	2(5) + 2(7) = 24	35
6	6	2(6) + 2(6) =24	36
7	5	2(7) + 2(5) = 24	35
8	4	2(8) + 2(4) = 24	32
9	3	2(9) + 2(3) = 24	27
10	2	2(10) + 2(2) = 24	20
11	1	2(11) + 2(1) = 24	11

- Graph the value of the width x and the area $12x - x^2$ in your notebook.

NEW Knowledge

A function is the relation between two magnitudes. Such a relation might be demonstrated through the use of tables, graphs or algebraic expressions. Tables and graphs give a broad vision of how variables relate; hence, it will be easy to understand phenomena changes.

- In a functional relationship, the value of the dependent variable changes with the value of the independent variable.
- Are we talking about a linear function? Explain your answer.
- What does it mean that there's a negative value? Explain your answer.
- How would the graphic representation be in the Cartesian Plane? Explain your answer.

Kells

SKILLS DEVELOPMENT

Reading skills: Interpreting statements.

Mathematizing skills: Understanding and using symbolic expressions, using constructs based on a formal system.

Reasoning skills: Discovering relations, making conjectures.

EVALUATION OF CONTENT

Students should be able to give a definition of functions in their own words.

Kells

Exercises and application

Solve the following problems individually. Once you have finished, compare your answers with a partner, describe your procedures, and present reasons to support your work.

A landowner and his parcel

1. A parcel is 50 m long and 30 m wide. The owner wants to enlarge it to raise his crops. He has the chance to buy part of the neighboring land and he wants his parcel to size 4 800 m^2. He wants to keep it a rectangle, and that's why he needs to add the same amount of land all around.
   - How long is each side of the land? How can you know that?
     60 × 80 because 60 × 80 = 4800.
   - What algebraic expression represents the problem? Write it down.
     $(30 + x)(50 + x) = 4800$
   - How big would the land be if the owner wanted a piece of land of 6 400 m^2? 8 000 m^2? 12 000 m^2? 19 500 m^2?
     90.64 × 70.64, 80 × 100, 120 × 100, 150 × 130
   - Make a table of values for the previous data and compare it with other students. Explain your procedure.

Area of the piece of land (in m^2)	Dimensions
6 400	90.64 × 70.64
8 000	80 × 100
12 000	120 × 100
19 500	150 × 130

A company issue

2. For a company to keep on working, it is necessary to cover minimum fixed expenses (FE). If such expenses correspond to the function:

$$FE\ (x) = f(x) = x^2 - 6x + 11$$

   - What's the minimum point of this function that corresponds to the minimum fixed expenses that have to be paid to keep operating?

→ Remember!

The minimum point or apex can also be obtained with the equation:

$$\left(\frac{-b}{2a} , \frac{4ac - b^2}{4a} \right)$$

Kells

SESSION INFORMATION

Week: 5

Sessions: 22 - 24

Expected Learning Outcome: Tabular and algebraic representation of quadratic variations, identified in different situations and phenomena in physics, biology, economics and other disciplines.

CONTENT DELIVERY

Start: Write on the board: *There's a program to reforest the area around the school. The trees to be planted grow 20 cm every year. We need to find when the trees will be 2.5 meters tall.* Then, help students find out the answer by guiding them relate the variables: $h(\text{age}) = \text{age} \times 20$.

Development: Now, students analyze the problem *A landowner and his parcel*. Get different students to respond questions in order to analyze the situation and answer the questions. You might ask students to develop the mathematical procedure on the board. Guide them step-by-step and elicit answers all the time from different students.

Closing: Students have to solve the second problem *A company issue* on their own.

SKILLS DEVELOPMENT

Reading skills: Interpreting mathematical information.

Mathematizing skills: Understanding and manipulating symbolic expressions.

Reasoning skills: Making inferences, providing a justification.

EVALUATION OF CONTENT

Students should be able to solve the second problem by themselves.

Kells

SESSION INFORMATION

Week: 5

Session: 25

Expected Learning Outcome: Tabular and algebraic representation of quadratic variations, identified in different situations and phenomena in physics, biology, economics and other disciplines.

CONTENT DELIVERY

Start: Check the answers to the problem *A company issue* by asking different students to develop the procedure on the board. Repeat any piece of information that you detect students cannot recognize or do easily.

Development: Students have to solve the problems *A rock in the well* and *The Lemur population on an island* individually.

Closing: Ask students to describe orally how to develop functions and when to use functions. If necessary, give students 5 to 10 more problems to solve using functions.

Homework: Students will need a coin, and dice.

- To solve the problem, make a table of values with x varying from 0 to 6. Follow the example below.

x	$f(x)$
x	$12 - x$
0	11
1	6
2	3
3	2
4	3
5	6
6	11

A rock in the well

3. When you throw a rock into a well it takes 20 seconds to reach the bottom.

- How deep is the well? How do you know that?

The well is 1962 m. You might want to use the formula $d = \frac{1}{2}gt^2$

- Consider that $y(t) = \frac{1}{2}gt^2$. Make a table of values for 15 s, 30 s, 45 s and 60 s.

FIG. 1.32 Lemur: any of various arboreal chiefly nocturnal prosimian primates that were formerly widespread but are now largely confined to Madagascar and that usually have a longish muzzle, large eyes, very soft woolly fur, and a long furry tail.

The lemur population on an island

4. On an island, there are 80 lemurs (Look at figure 1.32) and the population growth rate is given by the formula:

$$G(t) = -t^2 + 40t + 600$$

- How long will it take for the lemur colony to reach its highest population rate? Why is that?

20 years.

- Make a table of values for 5, 10, 15, 20, 25, 30, 40 and 50 years and share it with your partners.

t	Population (G)
5	775
10	900
15	975
20	1000
25	975
30	900
40	600
50	100

Kells

SKILLS DEVELOPMENT

Mathematizing skills: Transforming a real-world problem into a mathematical problem, using constructs based on formal systems, manipulating symbolic expressions.

Reasoning skills: Discovering relations, providing and checking a justification.

EVALUATION OF CONTENT

Check that students can analyze data and develop functions accordingly using the problems on pages 31 and 32.

Kells

Lesson 1.6
Probability. Complementary and Mutually Exclusive Events

Axis: Information Handling.

Topic: Probability Notions.

Pair work

Previous knowledge

In pairs, answer the following questions.

- When you toss a coin or throw a dice on several occasions and always under the same conditions, do you always get the same results? Why? What do we call those experiments?
- When you toss a coin, what are the possible results? And when you throw a dice, what are the possible results? What do we call those groups of possible results? What's the symbol that represents them?
- What do we call the fact of getting heads or tails when tossing a coin? What do we call the fact of getting a number from 1 to 6 when throwing a dice?

Pair work

Define the sampling space in the experiment below and in your notebook write it down.

- Throw two dices.
- Do the operations and describe how you got your answers.

FIG. 1.33 The Stock Market

In pairs, solve the experiment below. In your notebook, write the results down. Upon finishing, discuss the results with your teacher.

Experiment: each of the participants throws three times a dice.

- Define the sampling space.
- Write down the results you got.
- Are they independent events? Why?
- Are the results mutually exclusive? Support your answer.
- Share your answers with other partners and with your teacher.

FIG. 1.34 Services payment

NEW Knowledge

To calculate events probability that might be complex, it is possible to use a tool called *simulation*, in which you perform a simpler or more accessible experiment in such a way that it might be equivalent to the original situation. To estimate the required probabilities, it is necessary to repeat the equivalent experiment and calculate the relative frequency that is to be analyzed.

When simulating the problem, certain probability situations become simpler, because in real life it is complicated to control such phenomena as the movement in the Mexican Stock Market, for instance (figure 1. 33), the time it takes a person to make some service payments (figure 1.34) or even to determine whether it will rain or not (figure 1.35).

FIG. 1.35 Will it rain?

Kells

SESSION INFORMATION

Week: 6

Sessions: 26 - 28

Expected Learning Outcome: Probability scales knowledge. Characteristics analysis of complementary, mutually exclusive and independent events.

CONTENT DELIVERY

Start: Organize pairs. Have students toss the coin and the dice 10 times and write down the results they get making a table of values. Then, have different students read the questions in the section *Previous knowledge* and guide students to answer the questions correctly.

Development: Have students do the experiments and observations in pairs. Walk around the classroom to check that students are on task. Stop the activity when you see that three pairs have finished. Check answers in total class.

Closing: Students read the section *New knowledge*. Ask comprehension-check questions. Then, ask for any other application they can think of in which events probability is calculated.

SKILLS DEVELOPMENT

Reading skills: Interpreting statements.

Reasoning skills: Discovering relations, modeling with math.

Strategic skills: Experimenting.

Mathematizing skills: Transforming a real-world problem into a mathematical problem.

EVALUATION OF CONTENT

Students should be able to name events in which probability can be calculated, giving reasons why such events can be numbered.

Kells

SESSION INFORMATION

Week: 6

Sessions: 29, 30

Expected Learning Outcome: Probability scales knowledge. Characteristics analysis of complementary, mutually exclusive and independent events.

CONTENT DELIVERY

Session 29

Start: Organize trios. Have students analyze the problem The Insurance Agent.

Development: Students develop the problem.

Closing: Students compare results in total class and draw conclusions.

Session 30

Start: Organize trios. Have students answer problem 1. Elicit answers in total class.

Development: Have students answer problem 2. Elicit answers in total class, encourage discussion and have some students write the answers on the board.

Closing: Have students answer problem 3.

Homework: Students find out the definitions of: *mean*, *median*, *mode* and *range* in statistics.

Group activity

Analyze the situation and answer the questions.

The Insurance Agent

Joe Pendleton knows that every time he visits a client, he has a 20% chance (0.2 probability) of selling extended coverage insurance for a car, a 30% chance (0.3 probability) chance of selling a half coverage insurance; that is, that the insurance only covers damages but not medical care; 40% (0.4 probability) chance of selling a basic insurance; that is, a policy that just covers the person who has the accident, but the car repairs have to be fully paid by the driver and finally a 10% (0.1 probability) chance of selling nothing.

Look at the simulation table.

Product	Commission
Extended coverage insurance	US$250.00
Half coverage insurance	US$180.00
Basic insurance	US$100.00

Cut ten pieces of paper the same size and write down the products that the insurance agent sells:

- Two papers for extended coverage insurance.
- Three papers for half coverage insurance.
- Four papers for basic insurance.
- One paper that says "No sales".
- Put the papers in a bag and take out five; returning the paper that you take out so as not to alter the probability percentages with each one.
- Write down in your notebook the possible earnings that the agent might make with his five appointments.
- Compare your results with other teams and discuss them with your teacher.
- Write your conclusions on the board.

Exercises and application

Make a simulation for each exercise and write down the results in your notebook. When finishing, share the results with your group and your teacher.

1. A candy factory produces lollypops in three different flavors, in the proportion of 20% strawberry, 30% chocolate and 50% vanilla. What's the probability that when packing them at random in boxes of three per three, the three lollypops are all the same flavor?
2. A student responds a 10-question exam in which he only needs to answer True or False, but he's only sure of the answers he gives to five questions and he answers the other five questions at random. What's the probability that he gets a grade six or D?
3. Toss a coin five times, and write down the results you get. Calculate the following probabilities and indicate if they are independent events or mutually exclusive events. Do not forget to define the sample space.
    - Get heads two times.
    - Get tails five times.
    - The two possible events might occur.

Kells

SKILLS DEVELOPMENT

Mathematical skills: Using technical language and operations.

Reasoning skills: Reasoning quantitatively.

Verbal-linguistic skills: Explaining procedures.

EVALUATION OF CONTENT

Check students' procedures and results.

Kells

Lesson 1.7 Design of a Survey, Population Identification and Sampling

Axis: Information Handling.

Topic: Data Analysis and Representation.

Pair work

Previous knowledge

In Rio Frio, Estado de Puebla, Mr. Lawrence is in charge of fishponds with rainbow trout (figure 1.36). He wanted to analyze the total population in the ponds by checking two characteristics, the mass and size of the fish. He knew it would be risky and slow to get information from every single fish in the ponds because they can easily die when transported to be measured. So, one day he had the idea to register the data from fish that were taken out that very same day to make inferences from the total population in the ponds and the data he got is in the following table.

FIG. 1.36 The rainbow trout is commonly produced in natural ponds. It's a nutritious and delicious fish.

Number	Mass (g)	Size (cm)	Number	Mass (g)	Size (cm)
1	2 758	52	23	1 213	49
2	2 436	57	24	2 703	52
3	1 811	59	25	769	31
4	732	29	26	1 971	56
5	2 371	56	27	1 211	48
6	1 527	60	28	1 990	52
7	2 420	58	29	1 482	54
8	949	38	30	2 279	58
9	1 301	59	31	1 158	46
10	2 185	52	32	1 091	44
11	2 088	52	33	2 311	60
12	1 037	41	34	2 027	54
13	780	31	35	852	34
14	2 556	57	36	1 971	55
15	1 678	50	37	2 470	51
16	2 482	57	38	794	32
17	1 060	42	39	2 284	55
18	1 757	50	40	1 458	52
19	718	29	41	1 989	50
20	1 204	48	42	1 596	57
21	2 883	60	43	1 095	44
22	2 239	59	44	1 876	58

Kells

Look for information about the mean, median, mode and range in statistics.

SESSION INFORMATION

Week: 7

Session: 31

Expected Learning Outcome: Survey or experiment design a population study. Discussion over the ways to choose a sample. Data gathering from a sample and search of convenient presentation tools.

CONTENT DELIVERY

Start: Write on the board *mean, median, mode* and *range*. Ask different students to write the definition of each term on the board.

Development: Explain each term using the first five values in the table. Do it slowly, step by step and have different students do the operations on the board.

Closing: Students get the mean, median, mode and range of the following 10 values in the table.

SKILLS DEVELOPMENT

Reading skills: Interpreting statements.

Mathematical skills: Understanding and using symbolic expressions.

Reasoning skills: Reasoning quantitatively, discovering relations.

EVALUATION OF CONTENT

Students should be able to get the mean, median, mode and range of any table of values you exemplify.

Kells

SESSION INFORMATION

Week: 7

Sessions: 32 - 34

Expected Learning Outcome: Survey or experiment design a population study. Discussion over the ways to choose a sample. Data gathering from a sample and search of convenient presentation tools.

CONTENT DELIVERY

Start: Call for some students to write the formulas to get the mean, median, mode and range on the board.

Development: Have students analyze the data and complete each table. Upon finishing each exercise, call for different students to explain and develop the problem on the board. Make any necessary clarifications.

Closing: Check students' answers up to the first half of page 37. Have students do a population study about their school, hobbies, music or sports preferences. Ask students to design the survey they want to use, check it and have students perform the population study. They will have to get the mean, median, mode and range.

Homework: The following class, they will have to present the results of their study.

➔ Remember!

Statistics includes a number of techniques to analyze phenomena observations, data collection and information handling in order to obtain thorough knowledge and take better decisions.

Sampling is the technique to get data from a specific population group in order to get information about a general population group. This technique is useful to analyze the total population behavior or characteristics.

Pair work

Now, with the table of values on the previous page and your teacher's help complete the following tables and cake graph.

- Ask your teacher to help you remember how to obtain statistical data to fill out the following table:

Statistics	Mass (g)	Size (cm)
Mean	1717.29	48.43
Median	1971	52
Mode	1784	52
Range	2883 − 718 = 2165	60 − 29 = 31

- Now, Mr. Lawrence has a clear idea of the fishponds population, but he knows he can make this information even clearer. So, he made the following data grouping, called strata.
- Based upon previous knowledge, complete the following table:

Mass Strata	Frequency	Relative frequency	Percentage
1. 700 g to 1000 g	7	7 /total = 0.16	(Relative frequence) × 100 = 16%
2. 1 001 g to 1 500 g	11	0.25	25%
3. 1 501 g to 2 000 g	10	0.23	23%
4. 2 001 g to 2 500 g	12	0.27	27%
5. 2 501 g to +	4	0.09	9%
TOTAL	44	1.0	100%

- Make a bar graph with the frequency results (figure 1.37).

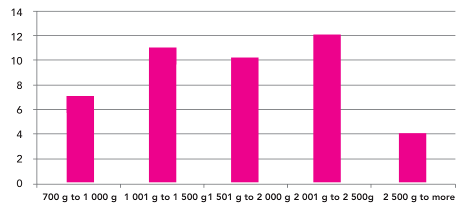

FIG. 1.37 Frequency.

- Make a bar graph with the relative frequency results (figure1.38).

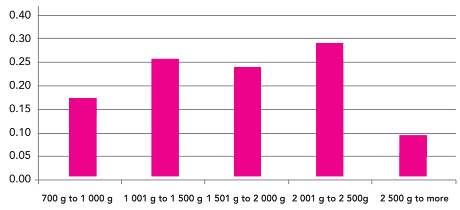

FIG. 1.38 Relative frequency.

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SKILLS DEVELOPMENT

Reading skills: Interpreting statements.

Mathematical skills: Understanding and using symbolic expressions.

Reasoning skills: Reasoning quantitatively, discovering relations.

EVALUATION OF CONTENT

Call for students' books and notebooks at random to check that students can follow the procedures.

Kells

- Complete the cake graph below writing the corresponding percentage in each wedge (figure1.39).

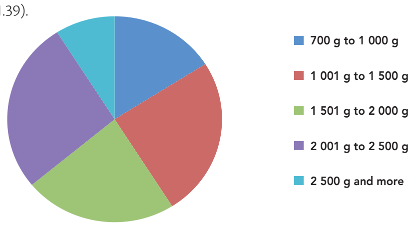


FIG. 1.39 Percentage.

Once you get the three graphs, answer the following questions about the **population** of rainbow trout:

- What population group predominates in the fishponds, according to the **sample**?
- What's the percentage that represents this stratum?
- What's the smallest population stratum living in the fishponds?
- Which are the most common fish to be caught?

- Now, follow the same procedure to analyze the data about the size of the fish.
- Take notes in your notebook and make the corresponding graphs for frequency, relative frequency and percentage.
- Answer the questions that Mr. Lawrence asked according to the results.
- Compare your results with those of other classmates.

Glossary

Population. It is the total group of people or other kind of elements that have common characteristics about which from whom certain information is required.

Sample. It is the subgroup of selected elements or people from a population that is to be analyzed in order to get data and make inferences about the total group.

Exercises and application

How well do you know your classmates?

1. Interview your classmates with the following questionnaire and analyze the necessary statistics to make inferences about all the students in the same school year in your school.
   - How old are you?
   - What's your gender?
   - How do you get to school?
2. Define age strata.
3. Classify gender with numbers: feminine = 1, masculine = 2
4. Write down the means to get to school and number them in order to build strata.

Kells

SESSION INFORMATION

Week: 7

Session: 35

Expected Learning Outcome: Survey or experiment design a population study. Discussion over the ways to choose a sample. Data gathering from a sample and search of convenient presentation tools.

CONTENT DELIVERY

Start: Check students' population study. Ask for a final report on their findings.

Development: Have students perform a major population study using the activity in the Exercises and application section.

Closing: Students will get the mean, median, mode and range and later they will present their findings.

SKILLS DEVELOPMENT

Reading skills: Interpreting statements.

Mathematical skills: Understanding and using symbolic expressions.

Reasoning skills: Reasoning quantitatively, discovering relations.

EVALUATION OF CONTENT

Check students' results and procedures to get the mean, median, mode and range.

Kells

SESSION INFORMATION

Week: 8

Sessions: 36 - 40

EVALUATION

CONTENT DELIVERY

Start: Students answer pages 38 and 39 prior to taking the unit assessment. Go through the answers in total class, guide students to remember core information of the unit.

Development: Students are to take the unit assessment. You can find it in this teacher's guide, pages 147 to 150, along with the answer key.

Closing: Check students' assessments, record scores and provide with feedback. You might want to use the attendance and evaluation formats that you can find in this teacher's guide, pages 175 and 176.

Evaluation

Solve the following problems individually. However, you can talk to other classmates about the procedures you will follow and check the necessary lessons before you actually answer.

1. The square of a number plus the same number is 240. What's the number? Explain why.

a) 12 **b)** 15 **c)** 25 **d)** 35

2. Analyze the information and answer the questions.

Mr. Brown bought a piece of land whose length triples its width and its area is 1 200 m². He will use the land to build a conference hall whose area will be 800 m².

a) How long is each side? 60 × 20 m.

b) Inside the piece of land there will be a cafeteria whose width triples its length and will be 75 m². How long will each side be in the cafeteria? 15 × 5 m

c) The bathroom will be a quadrangular room of 5 m each side. What's the area of the bathroom? 25 m²

d) What's the area that will be left free? 1 100 m²

e) What mathematical operations do you need to do in order to find the available space?

f) Which of the following algebraic expressions represents the area of the cafeteria and the bathroom?

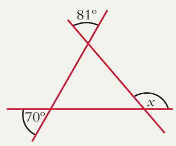
- $3x^2 = 105$
- $3x^2 + 5x - 100 = 0$
- $3x^2 = 75$
- $x^2 = 75$

g) If the conference hall is reduced 100 m2, is it possible to build a parking lot whose area doubles the one of the cafeteria? Explain your answer.

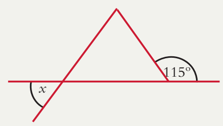
The area of the cafeteria and bathroom is subtracted from the size of the original piece of land, that is 1200 − 100 = 1 100 m²

3. In the following shapes, determine the length of the angle x you see:

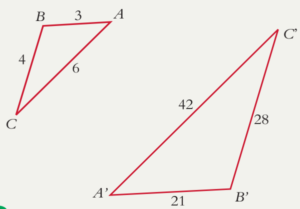
a) $x =$



b) $x =$



4. Determine if the following triangles are similar and explain why.



They are similar because $\frac{AB}{AB} = \frac{A_1C_1}{A_1B_1}$ and $\frac{AC}{BC} = \frac{A_1C_1}{B_1C_1}$

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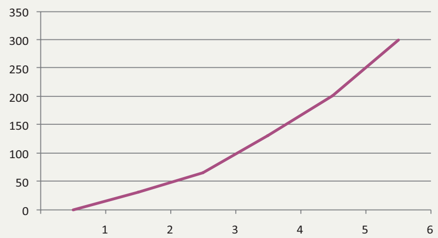
5. Which of the following pieces of information corresponds to direct proportionality?

a)

Time (h)	3	4	5	6
Distance (km)	240	320	400	480

b)



6. Mathias throws a stone vertically, it reaches its highest point in meters according to the time measured in seconds.
The function that describes such a situation is: $h\ (t) = -\ t^2 + 4t$
What's the highest point that the stone can get?
a) 2 m b) 3 m c) 4 m d) 5 m

7. How long does it take for the stone to reach the highest point?
a) 2 sec b) 3 sec c) 4 sec d) 5 sec

8. How high is the stone in the first second?
a) 1 m b) 2 m c) 3 m d) 3.5 m

9. How high is the stone in the third second?
a) 2 m b) 2.5 m c) 3 m d) 3.5 m

10. How well do you know your teachers?
Interview your teachers with the following questionnaire and analyze statistically the data you get to make inferences about all of the teachers in your school.

a) Classify two strata: Science = 1, Social Studies = 2
b) Separate them in age strata.
c) Classify gender by number.
d) Classify how they get to school by numbers to define strata.

- What studies do you have?
- How old are you?
- What's your gender?
- How do you get to school?

Answers will vary according to the data.

SESSION INFORMATION

Week: 8

Sessions: 36 - 40

EVALUATION

CONTENT DELIVERY

Start: Students answer pages 38 and 39 prior to taking the unit assessment. Go through the answers in total class, guide students to remember core information of the unit.

Development: Students are to take the unit assessment. You can find it in this teacher's guide, pages 147 to 150, along with the answer key.

Closing: Check students' assessments, record scores and provide with feedback. You might want to use the attendance and evaluation formats that you can find in this teacher's guide, pages 175 and 176.

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